

## Volume of a Solid of Revolution

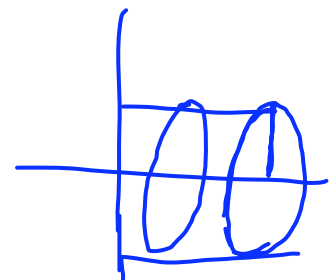
### Reminder: See

[http://college.cengage.com/mathematics/larson/calculus\\_analytic/7e/students/3dgraphs/ch06.html](http://college.cengage.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html)

A solid of revolution is a solid that is generated by revolving a plane region about a line that lies in the same plane as the region; the line is called the axis of revolution.

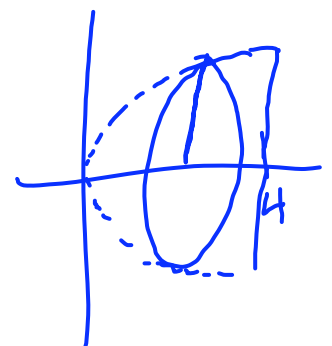
When revolving a region we take the region around the axis 360 degrees. Hence the cross section of a “slice” is always a circle. If the region is adjacent to the axis of revolution, then the resulting figure is solid – i.e. no “holes”. If any part of the region is not adjacent to the axis of revolution, then the resulting figure will have a “hole”. [Think of a donut!]

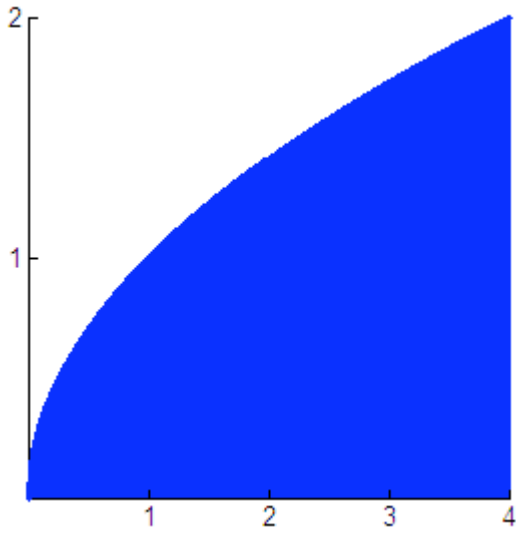
In most Calculus books the methods that we will be using are called “the Disk Method” and “the Washer Method” because our cross sections will look like a disk [a circle] or a washer.



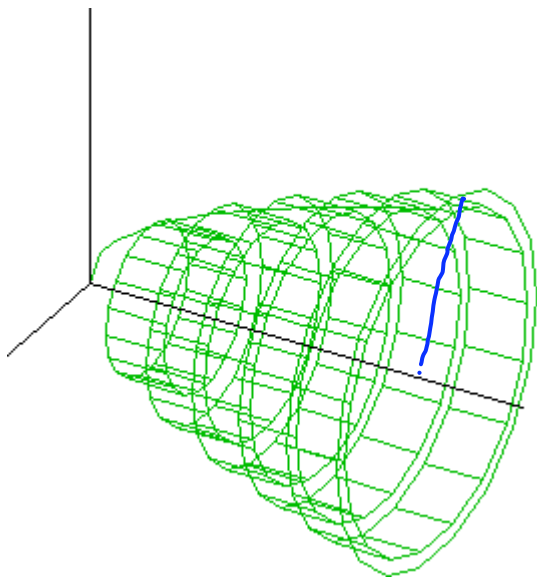
Let's see what this looks like:

Let our base region be the region enclosed by  $y = \sqrt{x}$ ,  
 $y = 0$ , and  $x = 4$

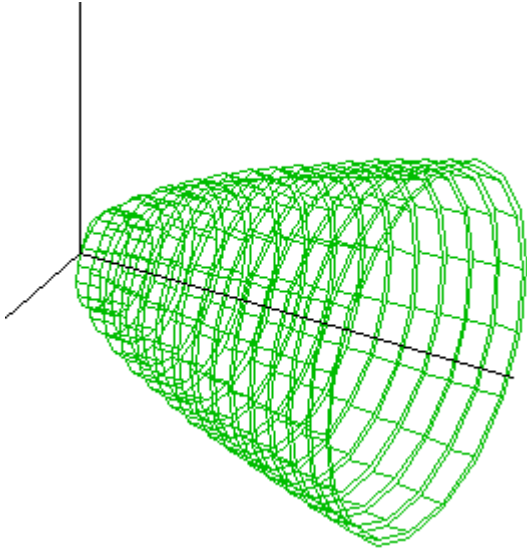




Go to and animate: <http://www.ies.co.jp/math/products/calc/applets/rotate/rotate.html>



Here is the solid with a few disks shown.



Here's with a lot of disks shown.

Volume = Area of a cross section \* height

Where the area of a cross section =  $\pi r^2$  and  $r = \sqrt{x}$

$$V = \int_0^4 \pi r^2 dx$$

depth of cross-section      cross-section

$$V = \pi \int_0^4 (\sqrt{x})^2 dx$$

$\pi r^2 = \text{area of a cross-section}$   
 $r = \sqrt{x}$

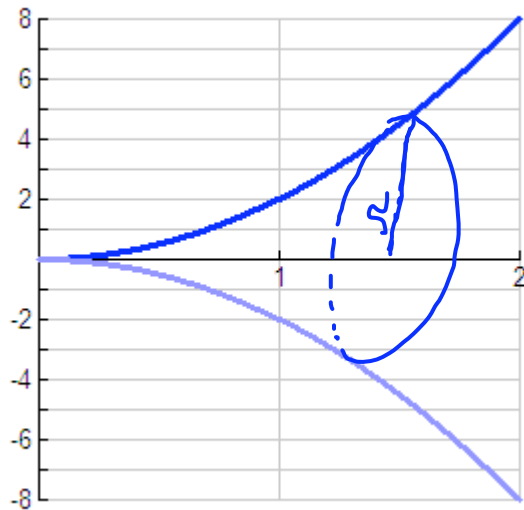
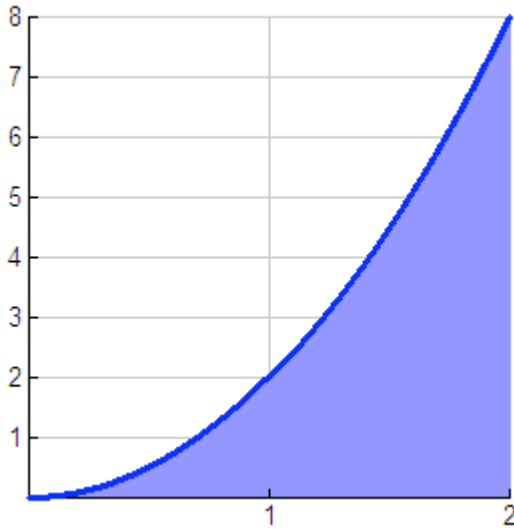
$$V = 8\pi(\text{units})^3$$

When revolving a region about the x-axis or y-axis, our cross sections will always be circles.

Let's try some:

Revolve the region defined by  $y = 2x^2$ ,  $y = 0$ ,  $x = 2$  about the x-axis

Here's the region:



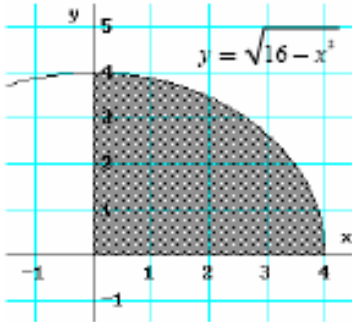
$$V = \int_0^2 \pi r^2 dx \text{ where } r = 2x^2$$

$$V = \pi \int_0^2 [2x^2]^2 dx$$

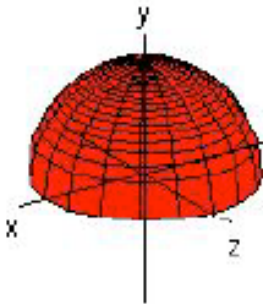
## Revolving a region about the y- axis

From: Mr. Leckie's website – <http://chaoticgolf.com>

Example: Find the volume of the solid formed by revolving the region about the y – axis. (Draw a representative rectangular strip)



$$y = \sqrt{16 - x^2}$$



$$y = \sqrt{16 - x^2}$$

$$y^2 = 16 - x^2$$

$$y^2 - 16 = -x^2$$

$$16 - y^2 = x^2$$

$$\sqrt{16 - y^2} = x$$

Volume [with vertical axis of revolution]

$$V = \pi \int_c^d [R(y)]^2 dy$$

$c, d$  are y- bounds, and  $R(y)$  is the radius in terms of  $y$

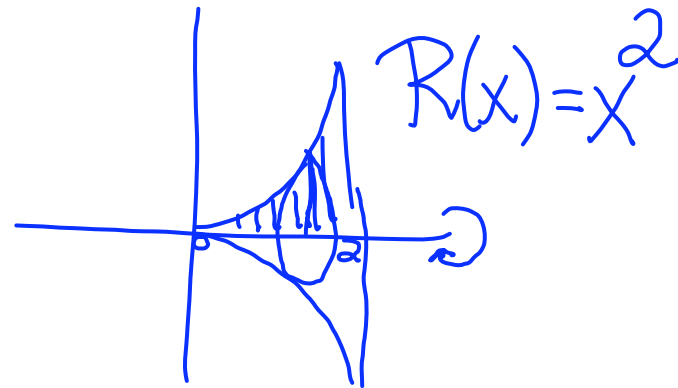
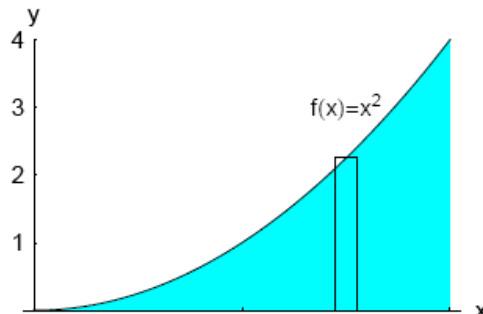
In this case:  $V = \pi \int_0^4 [\sqrt{16 - y^2}]^2 dy$

Let's consider one more:

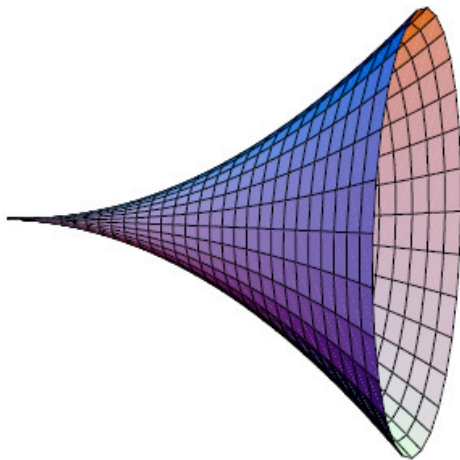
From: Mr. Zab's website

**Example 6:** Find the volume of the solid obtained when the region enclosed by the graph of  $f(x) = x^2$ , the x-axis, and the vertical lines  $x = 0$  and  $x = 2$  is revolved about the x-axis.

**Solution:** Look at the graph below.



When this region is revolved about the x-axis it will sweep out a region in space that looks like a volcano on its side (or maybe the end of a trumpet).



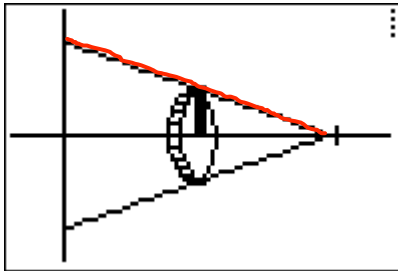
$$V = \pi \int_0^2 r^2 dx \text{ where } r = x^2$$

$$V = \pi \int_0^2 (x^2)^2 dx$$

Note: All of the solids formed today have no “holes”. In our next class we will explore how to find the volume when the solids has a “hole” in it [sort of like a doughnut].

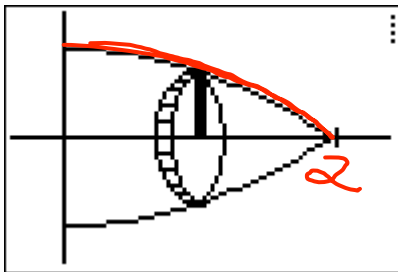
If time see page 463 Let's see what some of these look like.

1.  $y = -x + 1$



$$V = \pi \int_0^1 [-x+1]^2 dx$$

2.  $y = 4 - x^2$

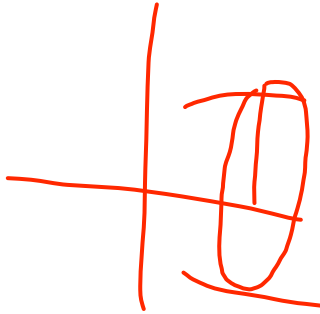
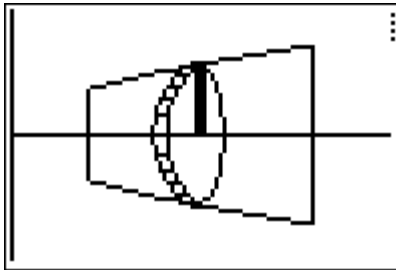


$$V = \pi \int_0^2 [4-x^2]^2 dx$$

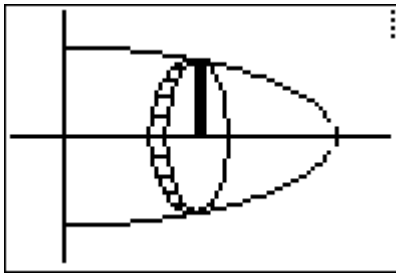


Homework: Read 7.2, do pages 463, 464 #3, 4, 7, 8, 9, 11A [set-up the integral and evaluate with your TI] Drawing a graph really helps. Here is what my graphs look like.

3.  $y = \sqrt{x}$

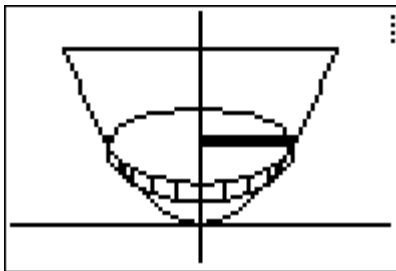


4.  $y = \sqrt{9 - x^2}$

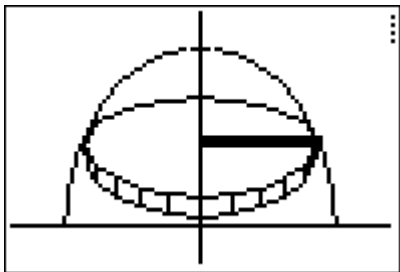


7.  $y = x^2$

Note: You will need to rewrite!



8.  $y = \sqrt{16 - x^2}$  Note: You will need to rewrite!



11a  $y = \sqrt{x}$

