AP PROBLEMS WITH $\ln x$ AND e^x

2000 AB6 [***non-calculator***]

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$

(b) Find the domain and range of the function found in part (a)

2003 AB4B [***non-calculator- so leave your answers in terms of e***]

A particle moves along the $x-$axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$

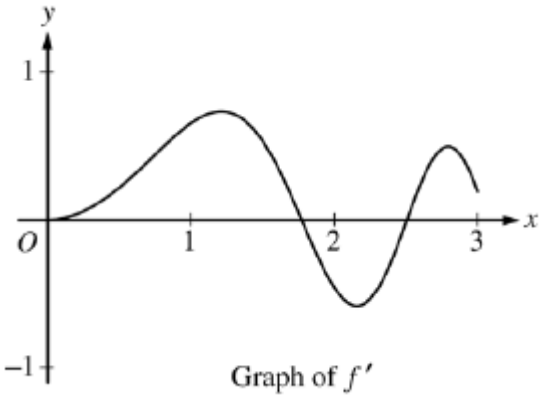
(a) Find the acceleration of the particle at time $t = 3$

(b) Is the speed of the particle increasing at time $t = 3$? Justify your answer [using Calculus!]

(c) Find all values of t at which the particle changes direction. Justify your answer [using Calculus!]

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

2006 AB2B [*calculator-friendly*]



Graph of f'

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{\frac{-x}{4}} \sin\left(x^2\right)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the open interval $1.7 < t < 1.9$. Justify.

(b) On the closed interval $0 \leq x \leq 3$, find the value of x at which f has an ***absolute maximum.*** Justify.

(c) Write an equation for line tangent to the graph of f at $x = 2$

A practice multiple-choice *[non-calculator]*

$$\int_1^e \left(\frac{x^2-1}{x}\right) dx =$$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

2008AB2 [***calculator-friendly***]

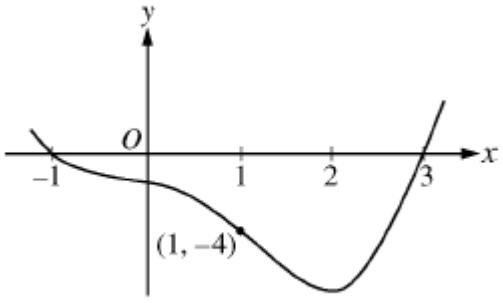
t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

[We already did parts a, b, and c]

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase ticket at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 500e^{\frac{-t}{2}}$ tickets per hour. Based on the model, how many tickets were sold by 3 p.m. $(t = 3)$, to the nearest number?

2009 AB5B ***[non-calculator]***



Graph of f'

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

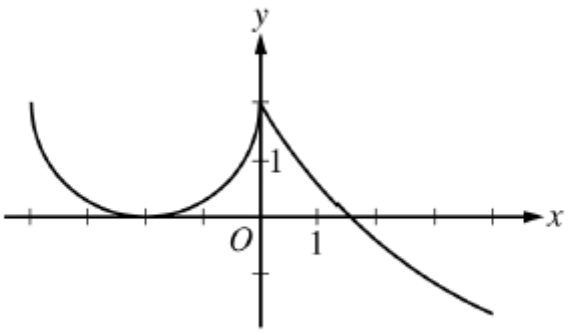
(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

2009 AB6 ***[non-calculator]***



Graph of f'

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & -4 \le x \le 0 \\ 5e^{-x/3} - 3 & 0 \,,\ x \le 4 \end{cases}$$

The graph of the continuous function f' is shown above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semi-circle, and $f(0) = 5$

(a) For $-4 < x < 4$ find all value(s) of x at which the graph of f has a point of inflection

(b) Find $f(-4)$ and $f(4)$

(c) For the closed interval $-4 \le x \le 4$, find the value of x at which f has an absolute maximum.

Another practice multiple-choice ***[non-calculator]***

. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

2008 AB2B [*calculator-friendly*]

For time $t \geq 0$, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by

$$g(x) = 0.05x\left(1 - e^{\frac{-x}{2}}\right)$$

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change ***with respect to time*** of the number of liters of gasoline used by the car when $t = 2$. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

1998 AB2 *[calculator-friendly]*

1998 AP Calculus AB Free-Response Questions

. Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

(c) What is the range of f?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

2001 AB2 *[calculator-friendly]*

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

Some more practice multiple-choice *[Calculator-friendly]*

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) –0.701
(B) –0.567
(C) –0.391
(D) –0.302
(E) –0.258

$\lim_{h\to 0} \dfrac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x) = \ln x$

(B) $f'(e)$, where $f(x) = \dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x+e)$

(E) $f'(0)$, where $f(x) = \ln x$