Some multiple-choice questions to practice:

5. 1985 AB42

$\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$ (D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$

(E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

2nd FTC

6. 1988 AB13

If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$

(A) $f(c)-f(0)$ (B) $|f(c)-f(0)|$ (C) $f(c)$ (D) $f(x)+c$

(E) $f''(c)-f''(0)$

$f(x)\Big|_0^c$

$= f(c)-f(0)$

8. 1988 BC14

If $F(x)=\int_1^{x^2}\sqrt{1+t^3}\,dt$, then $F'(x)=$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$ (D) $\sqrt{1+x^3}$

(E) $\int_1^{x^2}\frac{3t^2}{2\sqrt{1+t^3}}\,dt$

$F'(x) = \frac{d}{dx}\int_1^{x^2}\sqrt{1+t^3}\,dt$

$= \frac{d}{dx}(x^2)\sqrt{1+(x^2)^3}$

$= 2x\sqrt{1+x^6}$

9. 1993 AB41

$\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

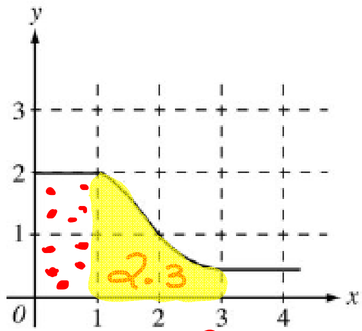
(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

2ND FTC

11. 1997 AB78

$$\int_0^3 F'(x)\,dx = F(x)\Big|_0^3 = F(3) - F(0) = 2 + 2.3$$

graph of f & $F'(x)$



CLUE

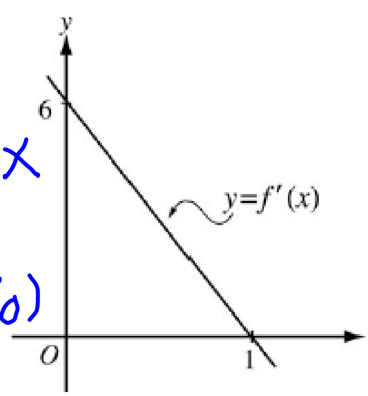
The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

$$\int_1^3 F'(x)\,dx = F(x)\Big|_1^3 = F(3) - F(1)$$

20. 2003 AB22

$$f(0) + \int_0^1 f'(x)\,dx = f(0) + f(1) - f(0) = 5 + 3$$



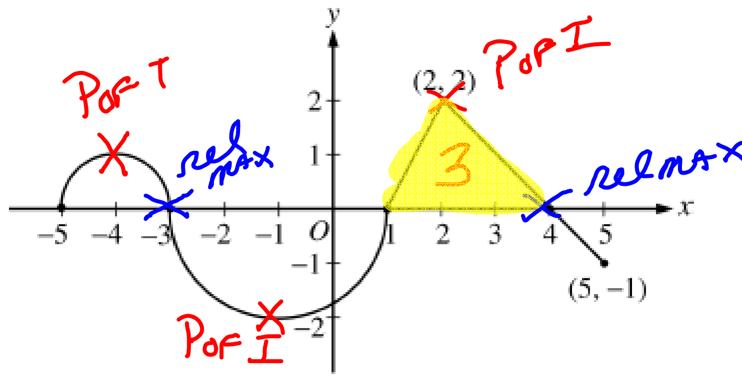
$$\int_0^1 f'(x)\,dx = f(x)\Big|_0^1 = f(1) - f(0) = \frac{1}{2}(1)(6)$$

The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

CLUE

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

8. 2007 AB4 [non-calculator]



Graph of f'

(c) Need f' to be increasing and $f'(x) > 0$

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

$\frown \; f'(x) > 0$

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

FIND f(4)

$$f(4) = f(1) + \int_1^4 f'(x)\,dx$$
$$= f(1) + f(x)\Big|_1^4$$
$$= f(1) + \underbrace{f(4) - f(1)}$$
$$= 3 + 3$$
$$= 6$$

For Homework:

2009 FTC and Accumulation Function [Chapter Four] **2010ftc.doc**

1. **2008 AB2 “Concert ticket problem” [calculator-friendly]**

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

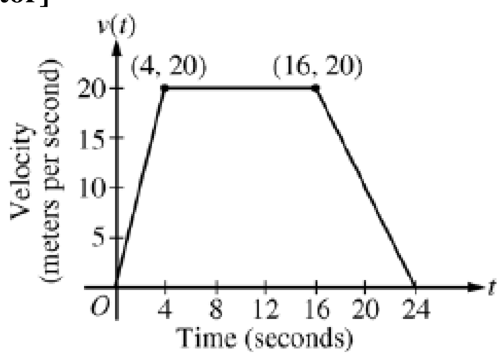
Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

3. **2005 AB5 [non-calculator]**



A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.