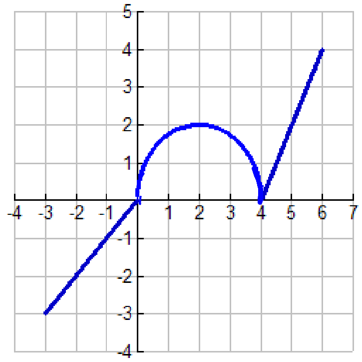
Chapter Four Free Response #1 Solution

Free Response #1



f is continuous on $[-3, 6]$

Graph of f

The graph of f shown above consists of a semi-circle and two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$

(A) Find $g(-2)$, $g'(-2)$, and $g''(-2)$

$$g(-2) = \int_0^{-2} f(t)\,dt$$

$$= -\int_{-2}^{0} f(t)\,dt$$

$$= -\frac{1}{2}(-2 \bullet 2)$$

$$= 2$$

$$g'(x) = \frac{d}{dx}\int_0^x f(t)\,dt$$

$$g'(x) = f(x) \text{ so, } g'(-2) = f(-2) = -2$$

$$g''(x) = f'(x) \text{ so, } g''(-2) = f'(-2) = \frac{f(0) - f(-2)}{0 - (-2)} = 1$$

(B) Let h be the function given by $h(x) = \int_{-3}^{x} f(t)\, dt$. Does h have a relative minimum, relative maximum, or neither at $x = 0$?

$$h'(x) = \frac{d}{dx}\int_{-3}^{x} f(t)\, dt \text{ So, } h'(x) = f(x)$$

Since the graph of $h'(x) = f(x)$ changes from negative to positive values at $x = 0$, then the graph of $h(x)$ has a relative minimum at $x = 0$.

BONUS Free Response [Optional]

(a) Find $g(6)$ and $h(6)$

$$g(6) = \int_{0}^{6} f(t)\, dt$$

$$= \frac{\pi}{2}\left(2^2\right) + \frac{1}{2}(2 \bullet 4)$$

$$h(6) = \int_{-3}^{6} f(t)\, dt$$

$$= \frac{1}{2}(-1)(3)(3) + 2\pi + 4$$