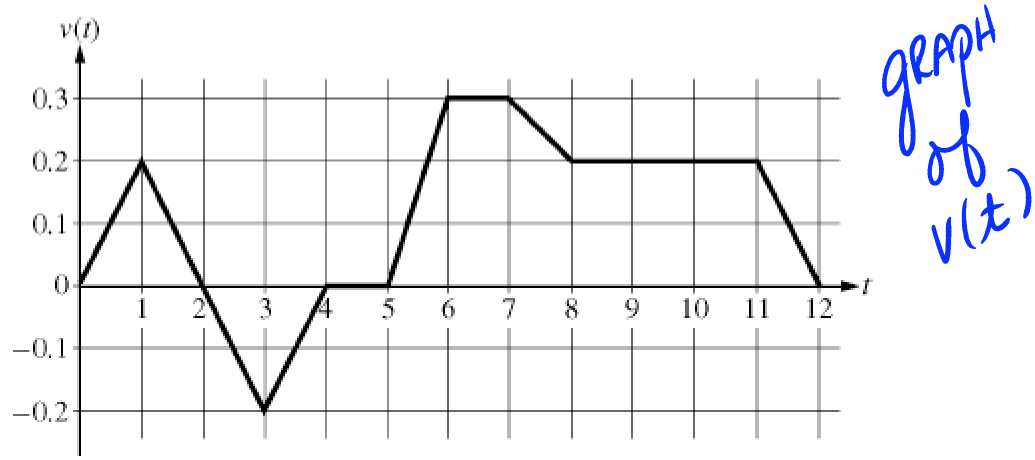


Pranav rides his bike along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, his velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Shortly after leaving home, Pranav realizes he left his calculus project at home, and he returns to get it. At what time does he turn around to go back home? Give a Calculus-based reason for your answer.

(b) How far did Pranav get before he turned around to go home? [Justify with Calculus]

(c) How long did it take Pranav to retrieve his Calculus project? [Indicate what time interval] (in other words, how long was he in the house looking for his project)

(d) Find all time intervals where the acceleration is equal to zero. [Justify with Calculus]

(e) Find the acceleration of Pranav's bicycle at $t = 7.5$ minutes. Indicate units of measure.

(f) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Pranav's trip. Find the value of $\int_0^{12} |v(t)|\, dt$

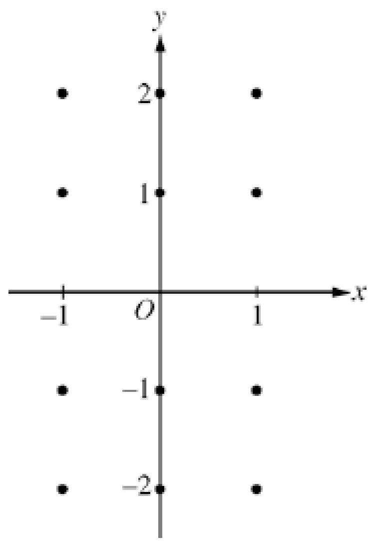
(g) Sir AB also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi t}{12}\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Find the acceleration of Sir AB's bicycle at $t = 7.5$ minutes.

(h) Who lives closer to school, Pranav or Sir AB. Show all the work the leads to your answer.

(i) Is the speed of Sir AB's bicycle increasing or decreasing at time $t = 7.5$ minutes. As always, justify with Calculus.

(2) Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$

On the axes provided below, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$

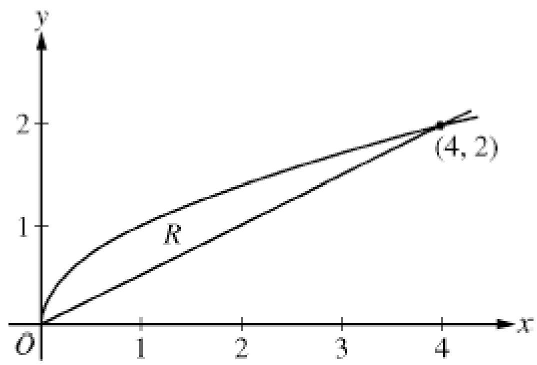
(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

(d) Find any point(s) at which the graph of f has a horizontal tangent. [If there are none, then be sure to justify with Calculus]

(e) Find any point(s) at which the graph of f has a vertical tangent. [If there are none, then be sure to justify with Calculus]

(f) State the domain of your solution to part (c)

(3)



Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semi-circles. Find the volume of this solid.

(d) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

(e) Find the volume of the solid formed when the region R is rotated about the x-axis.

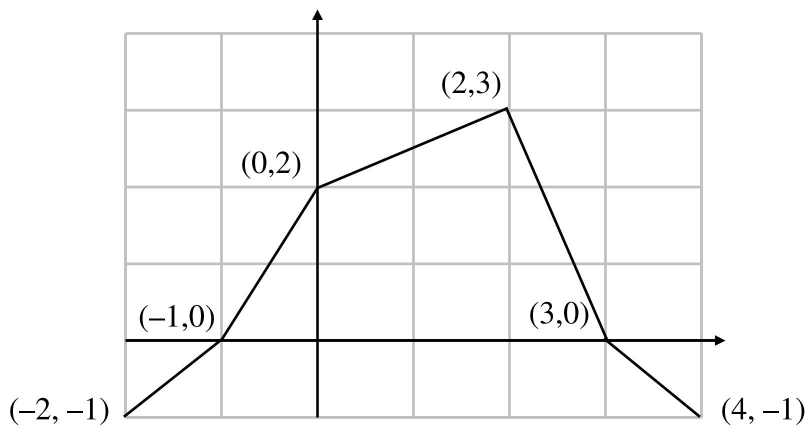
(f) Find the volume of the solid formed when the region R is rotated about the y-axis.

(g)

Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

(h) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the vertical line, $x = 4$

The Detective's Hat Function



The graph of the function f shown above is a piecewise continuous function defined on [–2, 4]. The graph of f consists of five line segments.

Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

This problem was written by Dixie Ross of for the Houston Area Calculus Teachers

Please do your work on a SEPARATE PIECE OF PAPER!

1. Find each of the following. Show all steps.

   (a) $g(-2)$ (b) $g(-1)$ (c) $g(0)$ (d) $g(2)$ (e) $g(3)$ (f) $g(4)$

2. Find each of the following. Show all steps.

   (a) $g'(-1)$ (b) $g'(0)$ (c) $g'(2)$ (d) $g'(4)$

3. Explain why g must be a continuous function on [–2, 4].

4. Write the equation for $g'(x)$ on the interval [0, 2].

5. Write the equation for the line tangent to g at $x = 1$.

6. Does $g''(0)$ exist? Explain your reasoning.

7. Will a point of inflection for g exist when $x = 0$? Explain your reasoning.

8. For what values of x in the open interval (–2, 4) is g increasing? Explain your reasoning.

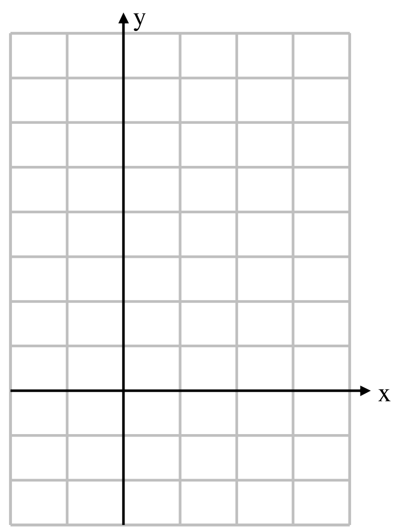
9. For what values of x in the open interval (–2, 4) is g decreasing? Justify fully.

10. For what values of x in the open interval (–2, 4) is g concave up? Explain your reasoning.

11. For what values of x in the open interval (–2, 4) is g concave down? Justify

12. Find the maximum and the minimum values of g on the closed interval [–2, 4]. Justify your answers.

13. On the axes provided, sketch the graph of function g on the closed interval [–2, 4].



For questions 14 – 15, let h be the function given by $h(x) = \int_{-2}^{x} f(t)\,dt$.

14. Find each of the following.

(a) $h(-2)$ (b) $h(-1)$ (c) $h(0)$ (d) $h(2)$ (e) $h(3)$ (f) $h(4)$

15. Find the following and explain your reasoning.

(a) $h'(-1)$ (b) $h'(0)$ (c) $h'(2)$ (d) $h'(4)$

16. $g(x) - h(x) = k$, where k is a constant. Find the value of k and explain your reasoning.