

Average Value of a Function

How do you usually find a mean [average]?

Do not confuse with **average rate of change [which is a slope]**.

Average value of a function is a value of $f(c)$.

If f is integrable [can be integrated] on $[a, b]$ then the average value of f is

$$\text{Average value} = \frac{1}{b-a} \int_a^b f(x) dx$$

You need to memorize this!!!!

Let's try one: $[a, b]$

Let $f(x) = 1 - x^2$ on $[-2, 2]$. Find the average value.

Our formula is $\frac{1}{b-a} \int_a^b f(x) dx$

$$\begin{aligned} \text{Average value} &= \frac{1}{2 - (-2)} \int_{-2}^2 [1 - x^2] dx \\ &= \frac{1}{4} \left[x - \frac{x^3}{3} \right]_{-2}^2 \\ &= \frac{1}{4} \left[\left(2 - \frac{8}{3} \right) - \left(-2 - \left(-\frac{8}{3} \right) \right) \right] \\ &= \frac{1}{4} \left[4 - \frac{16}{3} \right] \\ &= \frac{1}{4} \left[-\frac{4}{3} \right] \\ &= -\frac{1}{3} \end{aligned}$$

Find the average value: let $y = 3x^2$ on $[0, 3]$

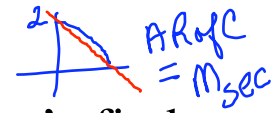
Our formula is Average value = $\frac{1}{b-a} \int_a^b f(x) dx$

$$\begin{aligned} & \frac{1}{3-0} \int_0^3 3x^2 dx \\ &= \frac{1}{3} \left[x^3 \right]_0^3 \\ &= \frac{1}{3} [27 - 0] \\ &= 9 \end{aligned}$$

Find the average value: let $y = 2 \cos x$ on $\left[0, \frac{\pi}{2}\right]$

Our formula is Average value = $\frac{1}{b-a} \int_a^b f(x) dx$

$$\begin{aligned} & \frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} 2 \cos x dx \\ &= \frac{1}{\frac{\pi}{2}} \left[2 \sin x \right]_0^{\frac{\pi}{2}} \\ &= \frac{1}{\frac{\pi}{2}} \left[2 \sin \frac{\pi}{2} - 2 \sin 0 \right] \\ &= \frac{1}{\frac{\pi}{2}} \cdot 2 \\ &= \frac{4}{\pi} \end{aligned}$$



Now that we have found the average value, let's find

the **average rate of change** for $y = 2\cos x$ on $\left[0, \frac{\pi}{2}\right]$

$$AR\ of\ C = \frac{f\left(\frac{\pi}{2}\right) - f(0)}{\frac{\pi}{2} - 0} = \frac{0 - 2}{\frac{\pi}{2}} = \frac{-4}{\pi}$$

Now let's look at some problems that ask for average value.

Traffic Flow Problem part (c)

What is the average value of the traffic flow over the time interval $[10, 15]$? Indicate units of measure.

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \quad \text{cars/minute}$$

$$\frac{1}{15-10} \int_{10}^{15} F(t) dt$$

$$= \frac{1}{5} [409.496] \approx 81.899 \frac{\text{cars}}{\text{minute}}$$

Thomasville, Oregon Problem

Traffic engineers will consider turn restrictions when

$L(t) \geq 150$ cars per hour. Find all values of t such that

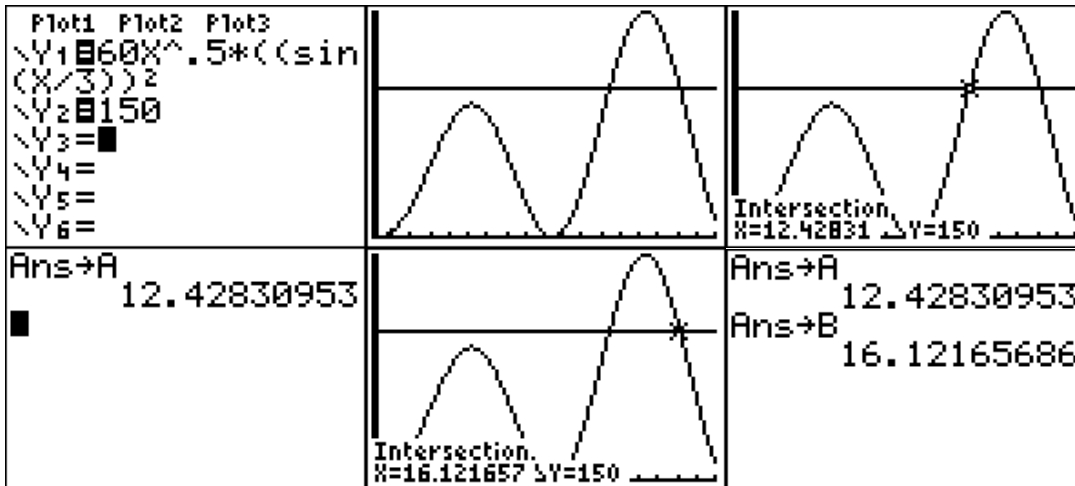
$L(t) \geq 150$ and compute the average value of L over this time interval. Indicate units of measure.

$$L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right) \quad \text{cars/hour}$$

$$L(t) = 150 \quad \text{at } t \approx 12.42831, 16.12166$$

$$\text{let } A = 12.42831, \text{ let } B = 16.12166$$

$$AV\ VALUE = \frac{1}{B-A} \int_A^B L(t) dt \approx 199.426 \text{ cars/hour}$$



Now we know our interval so we can find the average value using our handy-dandy formula

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fnInt(Y1,X,A,B)
736.549913
Ans/(B-A)
199.4261162

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Homework: page 291 #47 – 50 [Just find the average value!!!]

*SHOW your work
thank you*