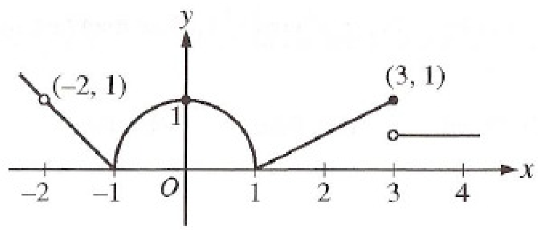

Graph of f

1. The graph of f is shown above.

(a) For how many values of c does $\lim_{x \to c} f(x) = 1$ [State the values of c]

$\lim_{x \to -2^-} f(x) = 1$ $\lim_{x \to -2^+} f(x) = 1$ Hence

$\lim_{x \to 0^-} f(x) = 1$ $\lim_{x \to 0^+} f(x) = 1$ $c = -2, 0$

$\lim_{x \to 3^-} f(x) = 1$ <u>but</u> $\lim_{x \to 3^+} f(x) \approx \frac{1}{2}$ but NOT at $x = 3$

(b) Let $g(x) = \int_{-1}^{x} f(t)\,dt$ and find the values of $g(-1), g'(-1)$, and $g(1)$

$g(-1) = \int_{-1}^{-1} f(t)\,dt = 0$

$g'(x) = \frac{d}{dx} \int_{-1}^{x} f(t)\,dt = f(x)$ so $g'(-1) = f(-1) = 0$

$g(1) = \int_{-1}^{1} f(t)\,dt = \frac{\pi}{2}(1^2) = \frac{\pi}{2}$

(c) Does the graph of g have any horizontal tangents? [Justify]

g has horizontal tangents at $x = \pm 1$ because $g'(x) = f(x) = 0$ at $x = \pm 1$

(d) Find any value(s) of x where the graph of g has a point of inflection. [Justify]

at $x = -1$ the graph of $g'(x) = f(x)$ changes from increasing to decreasing Hence g has a point of inflection at $x = -1$

at $x = 0$ the graph of $g'(x) = f(x)$ changes from decreasing to increasing Hence g has a point of inflection at $x = 0$

at $x = 1$ the graph of $g'(x) = f(x)$ changes from increasing to decreasing Hence g has a point of inflection at $x = 1$