Last Review for Chapter 6 and 7
2010lastch67review.doc

Solve the following separable differential equation with the initial condition $y(4)=0$

$$\frac{dy}{dx}=\frac{x}{-\sin y}$$

$$-\sin y\,dy = x\,dx$$

$$\int -\sin y\,dy = \int x\,dx$$

$$\cos y = \frac{x^2}{2}+C$$

$$\cos 0 = \frac{4^2}{2}+C \qquad C=-7$$

$$\cos y = \frac{x^2}{2}-7$$

$$\arccos(\cos y) = \arccos\left(\frac{x^2}{2}-7\right)$$

$$y = \arccos\left(\frac{x^2}{2}-7\right)$$

Find the general solution to the differential equation given below:

$$\frac{dy}{dx} = 4x^3 e^{-3y}$$

$e^{3y}dy = 4x^3 dx$

$\int e^{3y} dy = \int 4x^3 dx$

$\frac{1}{3} e^{3y} = x^4 + C,$

$e^{3y} = 3x^4 + C$

$\ln e^{3y} = \ln(3x^4 + C)$

$3y = \ln(3x^4 + C)$

$y = \frac{1}{3}\ln(3x^4 + C)$

Find the particular solution to the separable differential equation

$\frac{dy}{dx} = \frac{y-2}{2x}$ with the initial condition $\boxed{f(e) = 3}$

$$\frac{1}{y-2}\,dy = \frac{1}{2x}\,dx$$

$$\int \frac{1}{y-2}\,dy = \frac{1}{2}\int \frac{1}{x}\,dx$$

$$\ln|y-2| = \frac{1}{2}\ln|x| + C$$

$$e^{\ln|y-2|} = e^{\frac{1}{2}\ln|x|+C}$$

$$|y-2| = Ce^{\frac{1}{2}\ln|x|}$$

$$3-2 = Ce^{\frac{1}{2}\ln e}$$

$$C = \frac{1}{\sqrt{e}}$$

$$y-2 = \frac{1}{\sqrt{e}}\left[e^{\frac{1}{2}\ln x}\right]$$

$$y = 2 + \frac{1}{\sqrt{e}}\left[e^{\frac{1}{2}\ln x}\right]$$

$$|x| = \begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$$

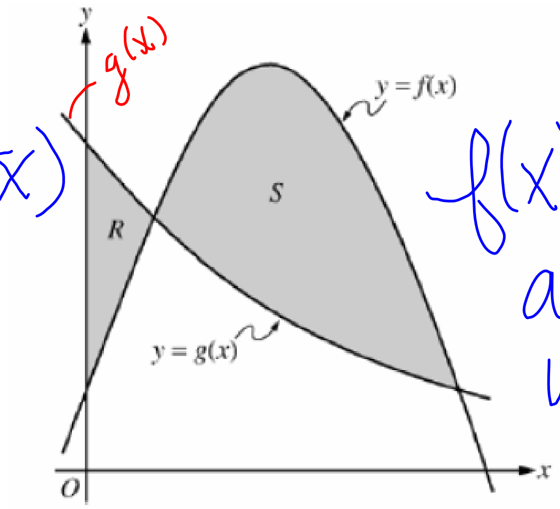
BUT IN THIS CASE $x \neq 0$

$$|y-2| = \begin{cases} 2-y, & y<2 \\ y-2, & y\geq 2 \end{cases}$$

BUT IN THIS CASE $y \neq 2$

$f(x) = \frac{1}{4} + \sin(\pi x)$

$g(x) = 4^{-x}$



$f(x) = g(x)$

at $x = A, B$

WHERE

$A \approx .1782180487$

$B = 1$

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g as shown above.

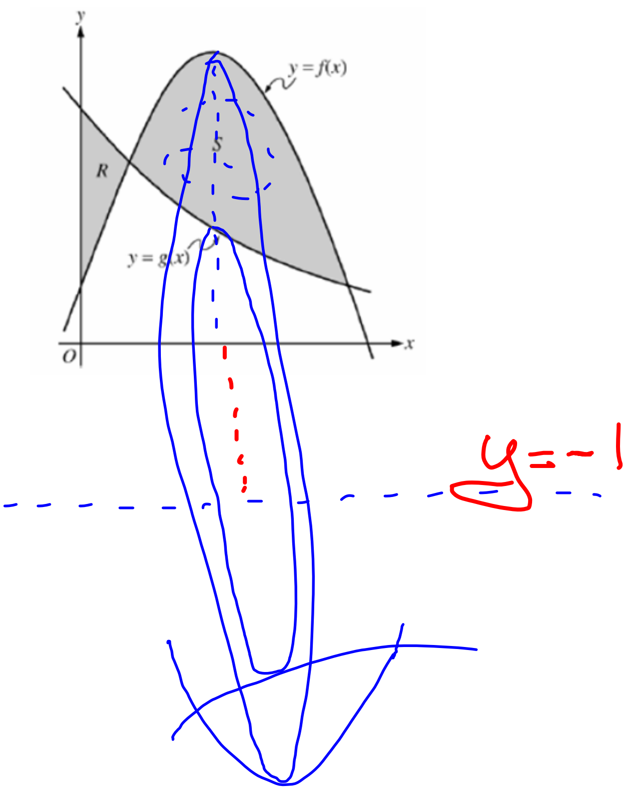
(a) Find the area of R

$$\text{AREA}_R = \int_0^A [g(x) - f(x)]\,dx \approx .0647$$

(b) Find the area of S

$$\text{AREA}_S = \int_A^1 [f(x) - g(x)]\,dx \approx .4103$$

(c) Find the volume of the solid generated when s is revolved about the horizontal line $y = -1$



$R(x) = 1 + f(x)$

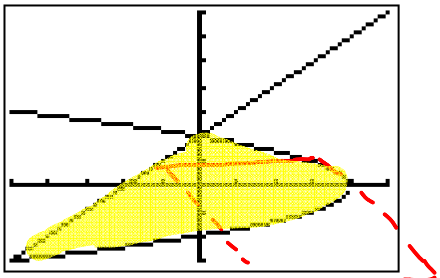
$r(x) = 1 + g(x)$

$V = \pi \int_{A} \left[(1+f(x))^2 - (1+g(x))^2 \right] dx$

$= \pi (1.451)$

Let R be the region formed by the ~~functions~~ CURVES on page 452 #13

$x = 4 - y^2$ and $x = y - 2$



$$4 - y^2 = y - 2$$
$$0 = y^2 + y - 6$$
$$0 = (y+3)(y-2)$$

Find the volume of the solid formed by using R as a base and the cross-section perpendicular to the $\boxed{y\text{-axis}}$ are squares.

$$A(y) = s^2$$
$$s = (4-y^2) - (y-2)$$

$$V = \int_{-3}^{2} \left[(4-y^2) - (y-2)\right]^2 dy$$

$$\approx \frac{625}{6}$$