More of the Second FTC

Slightly different:

$$F(x) = \int_{\frac{\pi}{2}}^{x^2} \cos t \; dt \qquad \text{Find } F'(x)$$

NOTE: We can't use a shortcut! Oh no!

$$F'(x) = \frac{d}{dx}\int_{\frac{\pi}{2}}^{x^2} \cos t \, dt$$

$$= \frac{d}{dx}\left[\sin t \Big|_{\frac{\pi}{2}}^{x^2}\right]$$

$$= \frac{d}{dx}\left[\sin\left(x^2\right) - \sin\left(\frac{\pi}{2}\right)\right]$$

$$= 2x\cos\left(x^2\right)$$ [Our old friend – The Chain Rule]

Isn't this the same as:

$$\frac{d}{dx}\int_{\frac{\pi}{2}}^{x^2} \cos t \, dt$$

$$= \cos\left(x^2\right) \bullet \frac{d}{dx}\left(x^2\right)$$

$$= 2x\cos\left(x^2\right)$$

OUR NEW SHORTCUT

REPLACE t WITH UPPER BOUND AND THEN MULTIPLY BY $\frac{d}{dx}$ OF UPPER BOUND

Another example:

Let $F(x)=\int_{3}^{3x} 2t\,dt$ Find $F'(x)$

$$F'(x)=\frac{d}{dx}\int_{3}^{3x} 2t\,dt$$

$$=\frac{d}{dx}\left[t^2\Big|_{3}^{3x}\right]$$

$$=\frac{d}{dx}\left[9x^2-9\right]$$

$$=18x$$

Once again, $F'(x)=2(\underline{3x})\bullet\frac{d}{dx}(3x)$

$$=(6x)(3)$$

Can you think of a shortcut?

Find $F'(x)$ for the given:

$$F(x)=\int_{5}^{x^3}\sec^2 t\;dt$$

$$F'(x)=\left[\sec^2(x^3)\right]\left[\frac{d}{dx}x^3\right]$$

$$=3x^2\sec^2(x^3)$$

$$F(x) = \int_{1}^{\cos x} 2t\, dt$$

$$F'(x) = (2\cos x)(-\sin x)$$
$$= -2\cos x \sin x$$

Now for the weirdest!

Let $F(x) = \int_{x}^{x^2} \cos t\, dt$ Find $F'(x)$

$$F'(x) = \frac{d}{dx}\int_{x}^{x^2} \cos t\, dt$$

$$= \frac{d}{dx}\left[\sin t \Big|_{x}^{x^2}\right]$$

$$= \frac{d}{dx}\left[\sin\left(x^2\right) - \sin(x)\right]$$

$$= 2x\cos\left(x^2\right) - \cos x$$

There are other ways of doing this but I think that we should stick to this way so that we don't get lost in a sea of equations.

Try the following using a shortcut if it is applicable:

Find $F'(x)$ for the following:

$$F(x)=\int_1^x \frac{t^3}{t^4+1}\,dt$$

$$F'(x) = \frac{x^3}{x^4+1}$$

$$F(x)=\int_1^{x^2} \frac{t^3}{t^4+1}\,dt$$

$$F'(x) = 2x\left(\frac{x^6}{x^8+1}\right) = \frac{2x^7}{x^8+1}$$

$$F(x)=\int_1^{\sin x} \sqrt[3]{t}\,dt$$

$$F'(x) = (\cos x)\sqrt[3]{\sin x}$$

$$F(x)=\int_0^{x^3} \sin\left(t^3\right)dt$$

$$F'(x) = 3x^2 \sin(x^9)$$

$$F(x) = \int_{\sin x}^{x^2} 2t\,dt \qquad \text{find } F'(x)$$

$$F'(x) = \frac{d}{dx}\int_{\sin x}^{x^2} 2t\,dt$$

$$= \frac{d}{dx}\left[t^2 \Big|_{\sin x}^{x^2} \right]$$

$$= \frac{d}{dx}\left[x^4 - \sin^2 x \right]$$

$$F'(x) = 4x^3 - 2\sin x \cos x$$

Homework: page 293 #81, 83, 85, 89, 90
Be sure to write the problem. You may use the shortcut IF it applies.

$$\frac{d}{dx} \sin^2 x$$

$$= \frac{d}{dx} u^2$$

$$= 2u \frac{du}{dx}$$

$$= 2 \sin x \cos x$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$