A Review of FTC so far

$$\int f'(x)\,dx = f(x) + C$$

$$\int_1^7 f'(x)\,dx = f(x)\Big|_1^7$$

$$= f(7) - f(1)$$

$$\int 2x\,dx = x^2 + C$$

$$\int_1^3 2x\,dx = x^2\Big|_1^3$$

$$= 3^2 - 1^2$$

$$= 8$$

$$\int \cos x\,dx = \sin x + C$$

$$\int_0^{\frac{\pi}{2}} \cos x\,dx = \sin x\Big|_0^{\frac{\pi}{2}}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin(0)$$

$$= 1$$

Now onto the Second FTC
Second Fundamental Theorem of Calculus

$$\int_0^x \cos t \, dt = \sin t \Big|_0^x$$

♪ Please look at the variables

$$= \sin x - \sin(0)$$
$$= \sin x$$

Now consider the following:
Order of Operations – do the integration first!

$$\frac{d}{dx}\int_0^x \cos t \, dt$$

see above

$$= \frac{d}{dx}(\sin x)$$
$$= \cos x$$

Hmm! Interesting!
Let's look at a different problem.:

$$\int_0^x 2t \, dt = t^2 \Big|_0^x$$

$$= x^2 - 0^2$$
$$= x^2$$

Now consider this:

$$\frac{d}{dx}\int_0^x 2t\,dt$$ see above

$$= \frac{d}{dx}\, x^2$$

$$= 2x$$

Very interesting! Is there a pattern?
Let's try one more to see.

$$\int_0^x (2t-5)\,dt = t^2 - 5t \Big|_0^x$$

$$= (x^2 - 5x) - (0^2 - (5)(0))$$

$$= x^2 - 5x$$

What about this now?

$$\frac{d}{dx}\int_0^x (2t-5)\,dt$$ see above

$$= \frac{d}{dx}\left(x^2 - 5x\right)$$

$$= 2x - 5$$

What seems to be happening?

Second FTC

If f is continuous on an open interval, I, containing a, then for every x in the interval $a \in \mathbb{R}$

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

a is not always equal to zero

♪ $\int_a^x f(t)\,dt$ is sometimes called an ***accumulation function*** OR SOME RATE of Δ because we are accumulating area. Also ♪ the upper bound must be x. If it is not x, then we will need to do some more work!

Let's try a few:

$$\frac{d}{dx}\int_1^x \sqrt[3]{t}\,dt$$

$$=\frac{d}{dx}\left[\frac{3}{4}t^{\frac{4}{3}}\Big|_1^x\right]$$

$$=\frac{d}{dx}\left[\frac{3}{4}x^{\frac{4}{3}}-\frac{3}{4}\right]$$

$$=x^{\frac{1}{3}} \text{ or } \sqrt[3]{x}$$

♪ $a = 1$ not zero

Does this verify the Second FTC? yes

$$\frac{d}{dx}\int_{\frac{\pi}{4}}^{x} \sec^2 t \; dt$$

$$= \frac{d}{dx}\left[\tan t \Big|_{\frac{\pi}{4}}^{x}\right]$$

$$= \frac{d}{dx}[\tan x - 1]$$

$$= \sec^2 x$$

Can we use a shortcut?

$$\frac{d}{dx}\int_{3}^{x} t^5 \; dt$$

$$= x^5$$

Beware of the following:

$$\frac{d}{dx}\int_{x}^{3} t^5 \; dt$$

What should we do?!

$$\frac{d}{dx}\left[-\int_{3}^{x} t^5 \, dt\right]$$

$$= -x^5$$

Let's try:

$$\frac{d}{dx}\int_2^x \left(\sin t + t^2\right)dt$$

$$= \sin x + x^2$$

$$\frac{d}{dx}\int_x^2 \left(\sin t + t^2\right) dt$$

$$= -\left[\frac{d}{dx}\int_2^x (\sin t + t^2)\,dt\right]$$

$$= -\left(\sin x + x^2\right)$$

$$\frac{d}{dx}\int_{10}^x \left(t^3 - 2t^2 + t\right) dt$$

$$= x^3 - 2x^2 + x$$

Ponder:

$$\frac{d}{dx}\underbrace{\int_{10}^{15} (x^3 - 2x^2 + x)\,dx}_{\text{NUMERICAL}} = 0$$

The Accumulation Function

Let $F(x) = \int_0^x (t-9)\,dt$

What would $F(0)$ be equal to?

$$F(0) = \int_0^0 (t-9)\,dt$$

$$F(0) = 0$$

What would $F(1)$ be equal to?

$$F(1) = \int_0^1 (t-9)\,dt$$

What would $F'(x)$ be equal to?

$$F'(x) = \frac{d}{dx}\int_0^x (t-9)\,dt$$

$$F'(x) = x-9$$

Homework:
Page 293 #75-80 [follow their directions because it will give us a good review of both the first the second FTC]