

2003 AB4B [non-calculator- so leave your answers in terms of  $e$ ]

$$e \approx 2.718$$

A particle moves along the  $x$ -axis with velocity at time  $t \geq 0$  given by  $v(t) = -1 + e^{1-t}$

(a) Find the acceleration of the particle at time  $t = 3$

$$a(t) = v'(t) = -e^{1-t}$$
$$a(3) = -e^{1-3} \text{ or } -\frac{1}{e^2}$$

(b) Is the speed of the particle increasing at time  $t = 3$ ? Justify your answer [using Calculus!]

$$v(3) = -1 + e^{1-3}$$

Since  $v(3) < 0$  and  $a(3) < 0$   
then speed is increasing at  $t = 3$

(c) Find all values of  $t$  at which the particle changes direction. Justify your answer [using Calculus!]

$$\text{let } v(t) = 0$$

$$0 = -1 + e^{1-t}$$

$$1 = e^{1-t}$$

$$t = 1$$

$$v(3) < 0$$

$$v(0) = -1 + e^{1-0} > 0$$

At  $t = 1$   $v(t)$  changes from positive to negative values hence the particle changes direction at  $t = 1$

(d) Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 3$ .

Please write down the following:

The total distance traveled on  $[a, b]$  is always equal to

$$\text{Total distance traveled} = \int_a^b |v(t)| dt$$

This is easy if the problem is a calculator-friendly problem but if it is not, then you need to determine when the velocity has negative and positive values.

$$\begin{aligned} \int_0^3 |v(t)| dt &= \int_0^1 v(t) dt - \int_1^3 v(t) dt \\ &= \int_0^1 [-1 + e^{1-t}] dt - \int_1^3 [-1 + e^{1-t}] dt \\ &= \left[ -t - e^{1-t} \right]_0^1 - \left[ -t - e^{1-t} \right]_1^3 \\ &= [(-1 - e^0) - (0 - e^1)] - [(-3 - e^{-2}) - (-1 - e^0)] \end{aligned}$$