Weird Chapter 2 Stuff **[2009weirdchapter2.doc]**

The Three Forms of Derivative as Limit [Or the Limit Definition of Derivative]

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ **Sometimes written with** $\Delta x = h$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$ **where** $c \in$ **real numbers**

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$ **where** $c \in$ **real numbers**

Rewrite the following as derivatives, then evaluate. Here is an example:

$\lim_{x \to 1} \frac{x^7 - 1}{x - 1}$ **This can be rewritten as** $\frac{d}{dx} x^7 \Big|_{x=1} = 7x^6 \Big|_{x=1} = 7$

1. $$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

2. $$\lim_{h \to 0} \frac{(10+h)^3 - 10^3}{h}$$

3. $$\lim_{x \to 10} \frac{x^3 - 1000}{x - 10}$$

4. $$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

5. $$\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{3} + h\right) - \cos\left(\frac{\pi}{3}\right)}{h}$$

6. $$\lim_{x \to 0} \frac{\cos x - 1}{x}$$

7. $$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

8. $$\lim_{x \to 5} \frac{-x^2 + 25}{x - 5}$$

9. $$\lim_{h \to 0} \frac{(2+h)^5 - 32}{h}$$

10. $$\lim_{h \to 0} \frac{\left(-(x+h)^3 + 2(x+h)\right) - \left(-x^3 + 2x\right)}{h}$$

11. Let $f(7)=0$, $f'(7)=14$, $g(7)=1$ and $g'(7)=\frac{1}{7}$.

Find $h'(7)$ if $h(x)=\frac{f(x)}{g(x)}$

12. From: http://www.wildstrom.com

Use the following table of values to compute the requested derivatives.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	–3	4	5	–1
2	5	1	3	7
3	–2	2	1	8
4	6	5	2	–3
5	9	3	4	–4

a. $H(x)=5f(x)-3g(x)$. Find $H'(1)$

b. $H(x)=\frac{f(x)}{g(x)}$. Find $H'(4)$

c. $H(x)=[g(x)]^5$. Find $H'(2)$

d. $H(x)=g(x^2-7)$. Find $H'(3)$

e. $H(x)=f(g(x))$. Find $H'(5)$

Problems #13 and #14 are from: http://cvc.org

13. The height of a cylinder with a radius of 4 cm is increasing at a rate of 2 cm per minute.

Find the rate of change of the volume of the cylinder with respect to time when the height is 10 centimeters. Note: $V = \pi r^2 h$

14. Two boats leave the same port at the same time with one boat traveling north at 15 knots per hour and the other boat traveling west at 12 knots per hour. How fast is the distance between the two boats changing after two hours?

15. Ms. McCleary has the unfortunate delight of teaching an AP Calculus class during the 1st period of the school day. For many reasons most of her students have a difficult time staying awake in class. Her evaluator took the following data one day while observing the class.

Elapsed class time [minutes]	5	10	15	20	25	30	35	40	45
Number of students who were awake	29	26	22	17	14	11	19	24	28

(a) The number of students awake is a function of time, t. Call this function $A(t)$. Using the data in the table, approximate $A'(15)$. What does this represent?

(b) Use your answer from part (a) to write the equation of the tangent line at $t = 15$.

(c) Use your answer from part (b) to estimate $A(17)$

And now for a slightly weird AP question

2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.