

## SOME “WEIRDER” AP FREE RESPONSE PROBLEMS

### 2007AB6 [non-calculator]

Let  $f$  be the function defined by  $f(x) = k\sqrt{x} - \ln x$  for  $x > 0$ , where  $k$  is a positive constant.

- Find  $f'(x)$  and  $f''(x)$ .
- For what value of the constant  $k$  does  $f$  have a critical point at  $x = 1$ ? For this value of  $k$ , determine whether  $f$  has a relative minimum, relative maximum, or neither at  $x = 1$ . Justify your answer.
- For a certain value of the constant  $k$ , the graph of  $f$  has a point of inflection on the  $x$ -axis. Find this value of  $k$ .

### 2007AB3 [calculator]

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

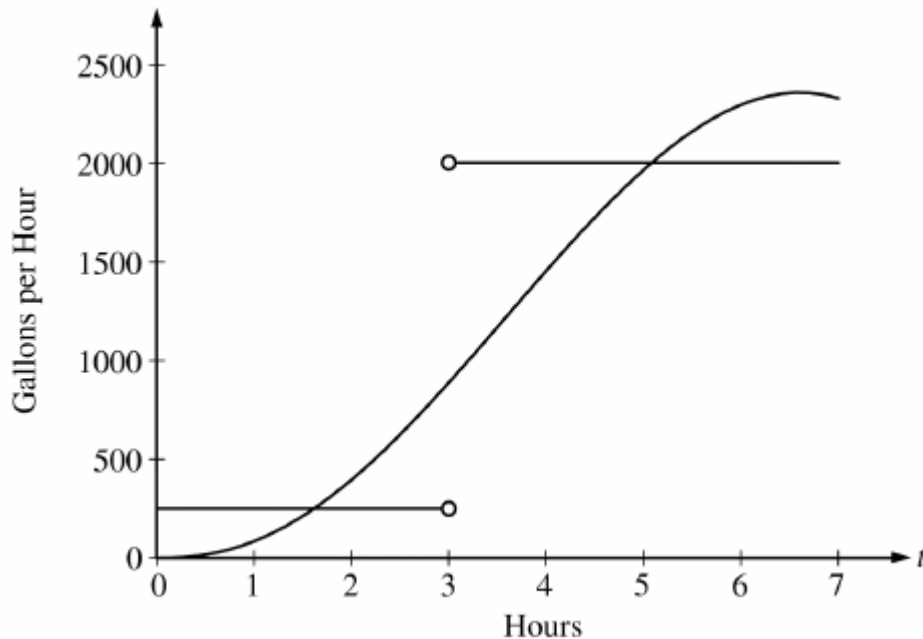
- The functions  $f$  and  $g$  are differentiable for all real numbers, and  $g$  is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of  $x$ . The function  $h$  is given by  $h(x) = f(g(x)) - 6$ .
- Explain why there must be a value  $r$  for  $1 < r < 3$  such that  $h(r) = -5$ .
  - Explain why there must be a value  $c$  for  $1 < c < 3$  such that  $h'(c) = -5$ .
  - Let  $w$  be the function given by  $w(x) = \int_1^{g(x)} f(t) dt$ . Find the value of  $w'(3)$ .
  - If  $g^{-1}$  is the inverse function of  $g$ , write an equation for the line tangent to the graph of  $y = g^{-1}(x)$  at  $x = 2$ .

### 2008AB4B [non-calculator]

The functions  $f$  and  $g$  are given by  $f(x) = \int_0^{3x} \sqrt{4+t^2} dt$  and  $g(x) = f(\sin x)$ .

- Find  $f'(x)$  and  $g'(x)$ .
- Write an equation for the line tangent to the graph of  $y = g(x)$  at  $x = \pi$ .
- Write, but do not evaluate, an integral expression that represents the maximum value of  $g$  on the interval  $0 \leq x \leq \pi$ . Justify your answer.

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2007AB2



The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is  $f(t) = 100t^2 \sin(\sqrt{t})$  gallons per hour for  $0 \leq t \leq 7$ .
- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.
- (b) For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.