Volumes with Washers

What happens when the area we are revolving about an axis is not adjacent to the axis of revolution?

We will get a "hole"! Note the difference between the two solids of revolution below.

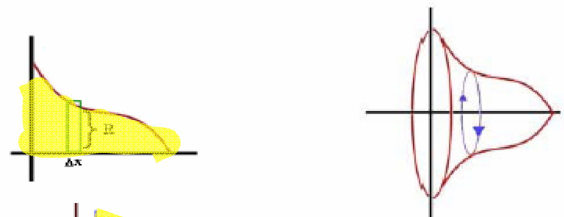
http://www.epcc.edu/Student/Tutorial/Mathcenter/handouts/calculus/volumeshandout.pdf

Volumes By Disks and Washers

When solving problems involving rotating an area about an axis, the disk or washer methods may be used. Both are interchangeable in most cases, but sometimes it may be easier to use one method over another. After setting up your integral, the volume follows the same integration rules of other problems.

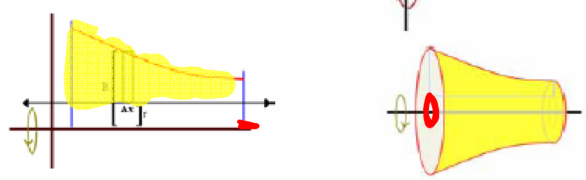
Volumes by Disks

$$V = \int_a^b \pi\,[f(x)]^2\,dx$$



Volumes by Washers

$$V = \int_a^b \pi\,[(f(x))^2 - (g(x))^2]\,dx$$



The easiest way to view these types of solids is to admire the animations at:

http://mathdemos.gcsu.edu/mathdemos/washermethod/gallery/gallery.html

Steps:

1. Graph the region that you will be revolving
2. Draw a representative radius so that you can clearly identify the $R(x)$, the outer radius, and $r(x)$, the inner radius

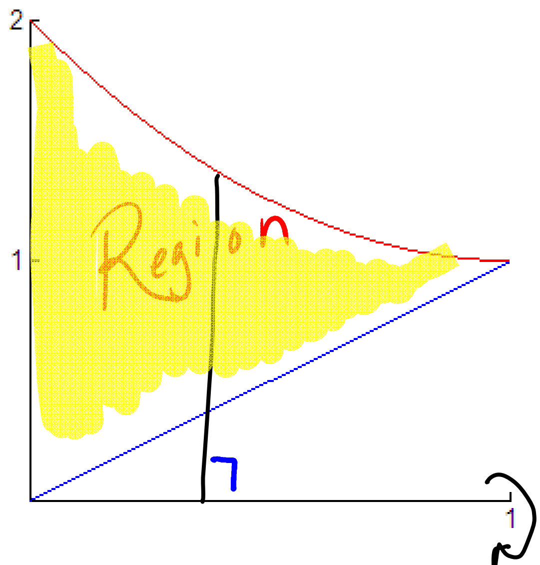
   whole - hole

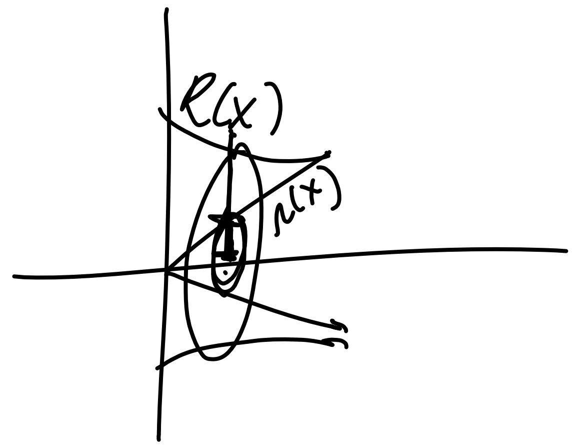
3. Use the formula $V = \pi \int_a^b \left[\left(\underset{\text{outer}}{R(x)}\right)^2 - \left(\underset{\text{inner}}{r(x)}\right)^2 \right] dx$

   about the x-axis

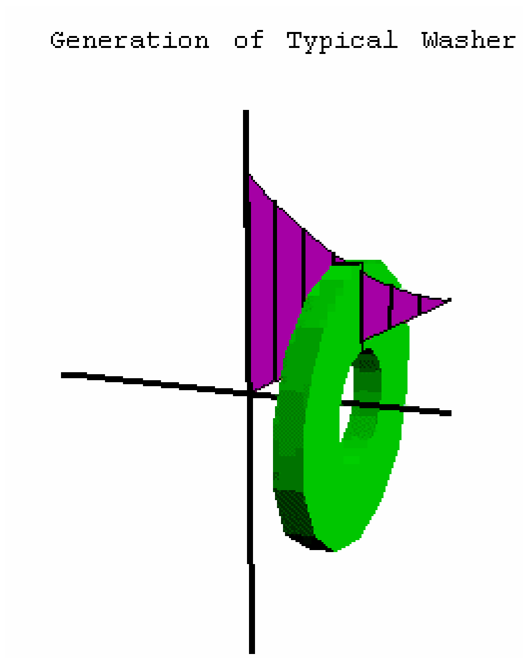
Do NOT use the chucklehead set-up!!

Example 1: Find the volume of the solid of the region bounded by $y = x$, $x = 0$, and $y = (x-1)^2 + 1$, which is revolved about the x-axis.





Here's the region with a representative radius drawn in.
Let's look at an animation of a rotation.
http://mathdemos.gcsu.edu/mathdemos/washermethod/gallery/gallery.html



$R(x) = (x-1)^2 + 1$

$r(x) = x$

R(x) INTERSECTS r(x)

$(x-1)^2 + 1 = x$

$x^2 - 2x + 1 + 1 = x$

$x^2 - 3x + 2 = 0$

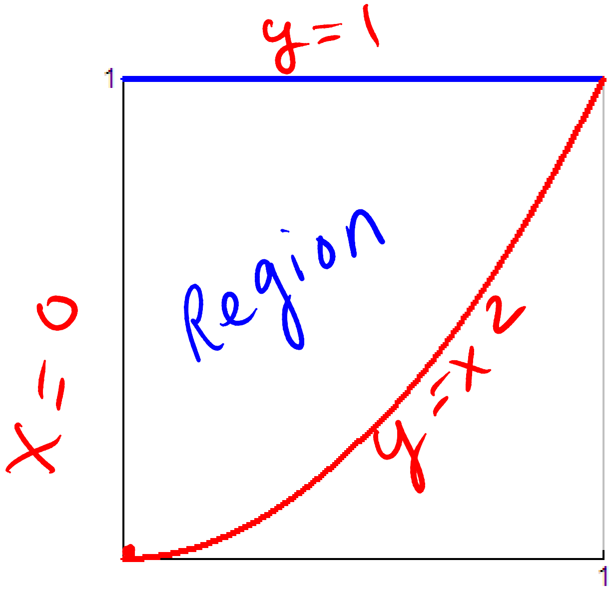
$(x-2)(x-1) = 0$

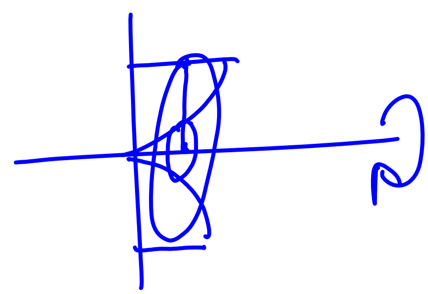
$$V = \pi \int_0^1 \left[(R(x))^2 - (r(x))^2 \right] dx$$

$$V = \pi \int_0^1 \left[(x^2 - 2x + 2)^2 - (x)^2 \right] dx$$

$V = \frac{23}{15}\pi$ CUBIC UNITS

Example 2: Find the volume of the solid whose region is bounded by $\underline{y = 1}$, $x = 0$, $y = x^2$ revolved about the x-axis.



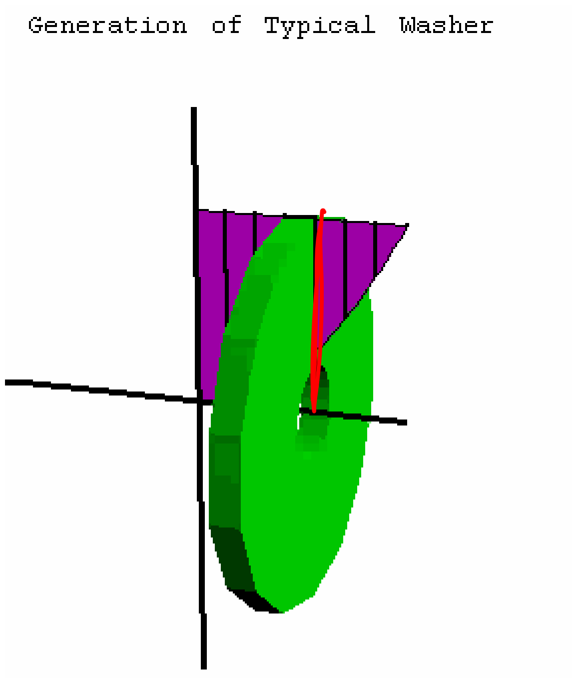


$R(x) = 1$

$r(x) = x^2$

Let's look at another animation:

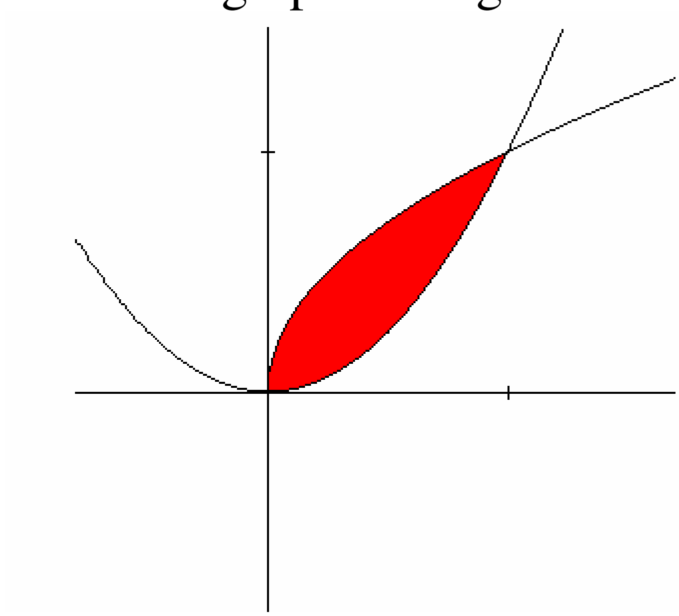
http://mathdemos.gcsu.edu/mathdemos/washermethod/gallery/gallery.html



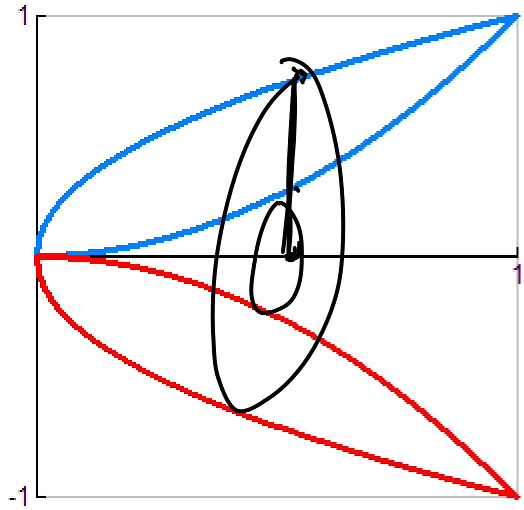
$$V = \pi \int_0^1 \left[1^2 - (x^2)^2\right] dx$$

Let's consider the region bounded by $y = \sqrt{x}$, $y = x^2$ rotated about the x-axis

First let's graph the region



If rotated about the x-axis, then we can visualize a side view as:



Draw in a representative radius and find the outer radius and the inner radius

$R(x) = \sqrt{x}$
$r(x) = x^2$

Fill in the values for our formula:

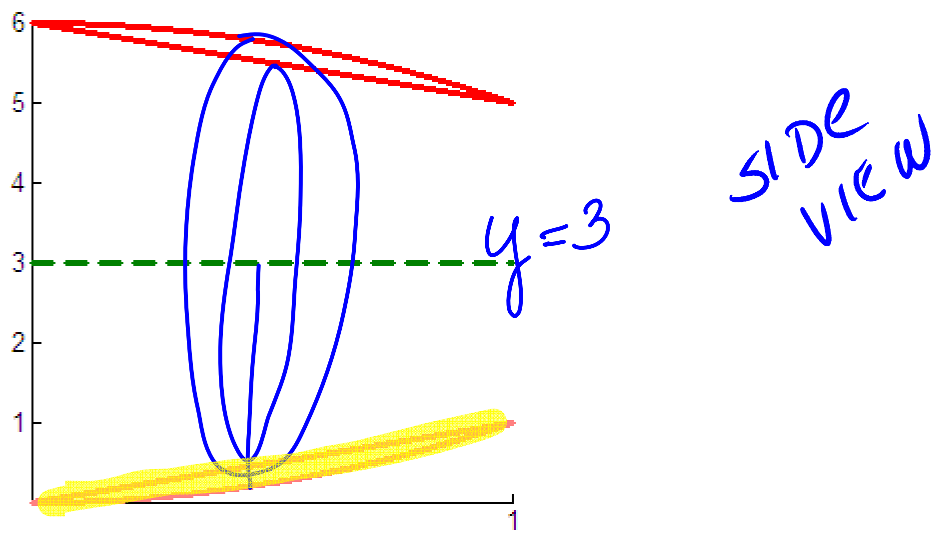
$$V = \pi \int_a^b \left[(R(x))^2 - (r(x))^2 \right] dx$$

$$V = \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2 \right] dx$$

Now find the volume $= \frac{3}{10}\pi$ cubic units

Now let's consider revolving about a line that is NOT an axis

The region bounded by the curves $y = x$ and $y = x^2$ is rotated a bout the line $y = 3$. Compute the volume of the resulting solid.



Draw a representative radius to help determine the inner radius and outer radius

$R(x) = 3 - x^2$ BIG RADIUS

$r(x) = 3 - x$ LITTLE RADIUS

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

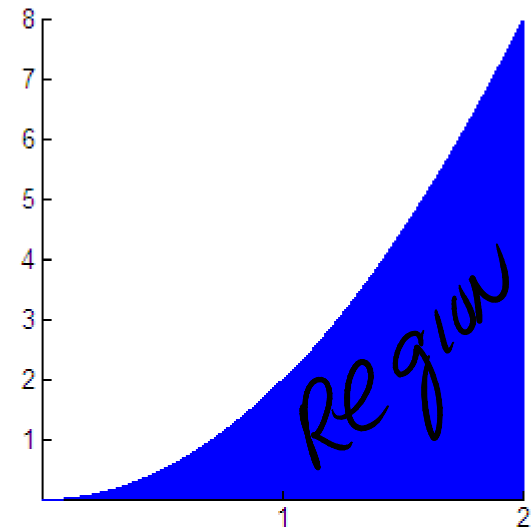
Now set up the integral

$V = \pi \int_a^b \left[(R(x))^2 - (r(x))^2 \right] dx$ and find the volume

$$V = \pi \int_0^1 \left[(\underbrace{3 - x^2}_{R(x)})^2 - (\underbrace{3 - x}_{r(x)})^2 \right] dx$$

Let's set up some problems. See page 463 #12

$y = 2x^2,\ y = 0,\ x = 2$



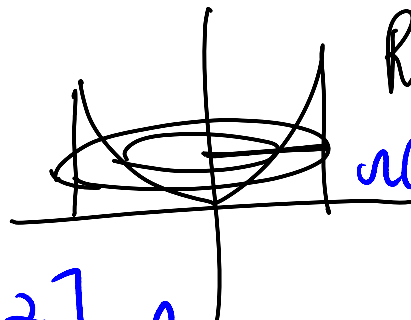
Here's the region:

$y = 2x^2$

$\sqrt{\frac{y}{2}} = x$

(a) About the y-axis

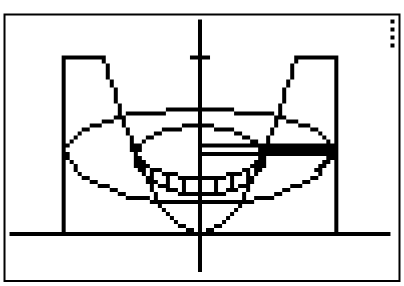
We'll need $R(y)$ and $r(y)$



$R(y) = 2$

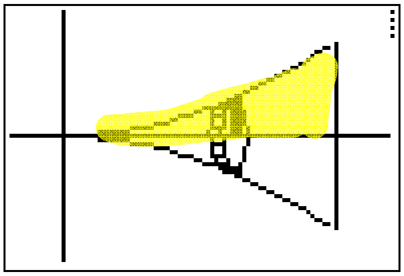
$r(y) = \sqrt{\frac{y}{2}}$

$$V = \pi \int_0^8 \left[2^2 - \left(\sqrt{\frac{y}{2}}\right)^2\right] dy$$



(b) About the x-axis

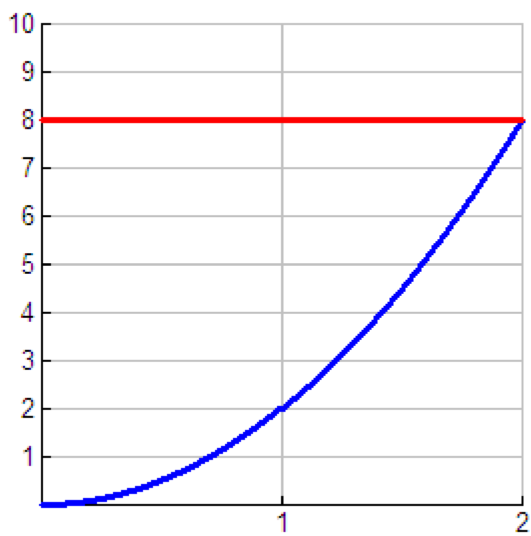
We'll need $R(x)$ [and $r(x)=0$]

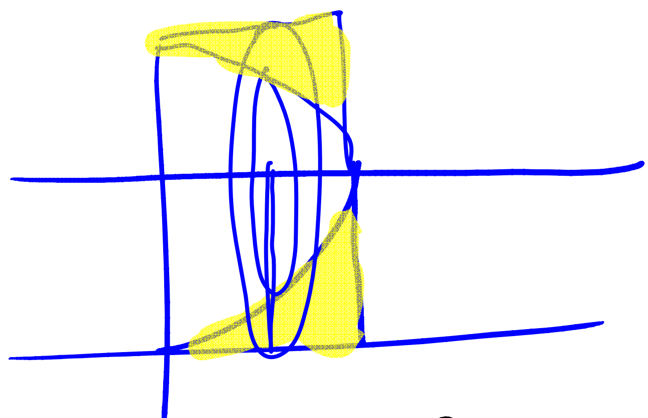


no holes

$$V = \pi \int_0^2 \left[(2x^2)^2 \right] dx$$

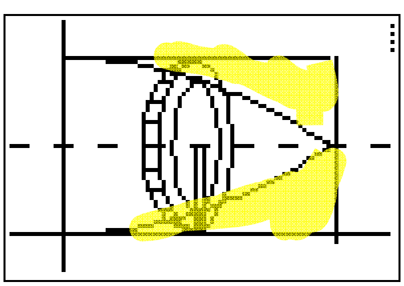
(c) About the line y=8 We'll need $R(x)$ and $r(x)$





$R(x) = 8$

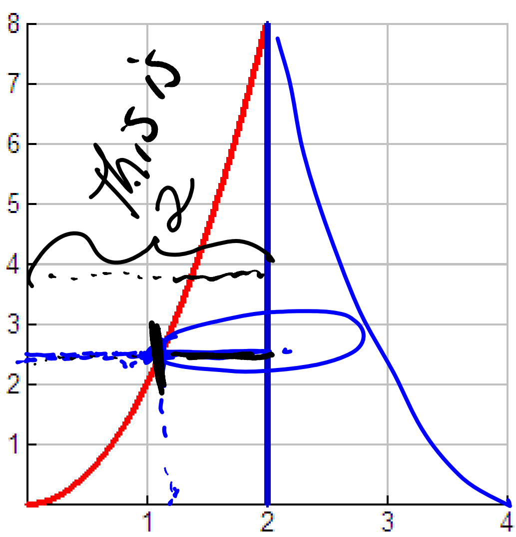
$r(x) = 8 - 2x^2$



$$V = \pi \int_0^2 \left[8^2 - (8 - 2x^2)^2 \right] dx$$

$y = 2x^2$
$x = \sqrt{\frac{y}{2}}$

(d) About the line x = 2 We'll need $R(y)$ $[r(y) = 0]$



$R(y) = 2 - \sqrt{\frac{y}{2}}$?

$$V = \pi \int_0^8 \left[2 - \sqrt{\frac{y}{2}}\right]^2 dy$$

Homework: Finish the rest of the problems on the "Worksheet: Volumes" handout

#6 about
$y = -1$