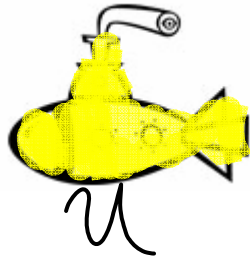


Integration by Substitution [Anti-Chain Rule]

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

$$\int f'(g(x))g'(x)dx = f(g(x)) + C$$

You should look for a composite function! Just like the Chain Rule, we'll need to find a u .



This is the u-sub!

Composite Function	What is u ?	What is $\frac{du}{dx}$?
$2\sin(2x)$	$2x$	2
$(3x^7 + 11)^5 (21x^6)$	$3x^7 + 11$	$21x^6$
$\sqrt{4x^5 - 5x} (20x^4 - 5)$	$4x^5 - 5x$	$20x^4 - 5$
$\tan^2 x \sec^2 x$	$\tan x$	$\sec^2 x$

Here is our first example:

$$\int \underbrace{(x^2 + 5)}^{10} (2x) dx$$

What is the composite function? $(x^2 + 5)^{10}$

What is the “inside” function $[g(x)]$?

Call it u

Find du

$$\begin{aligned} u &= (x^2 + 5) \\ \frac{du}{dx} &= 2x \\ du &= 2x dx \end{aligned}$$

Rewrite our original integral using all of the u-substitutions

$$\int \underbrace{(x^2 + 5)}^{10} \underbrace{(2x)} dx \text{ becomes } \int u^{10} du$$

Now solve in terms of u

$$= \frac{u^{11}}{11} + C \quad \text{Now rewrite solution in terms of } x$$

$$\int (x^2 + 5)^{10} (2x) dx = \frac{(x^2 + 5)^{11}}{11} + C$$

Our second example:

$$\int \underline{2} \sin(\underline{2x}) \underline{dx}$$
$$u = 2x$$
$$\frac{du}{dx} = 2$$
$$du = 2 dx$$

Rewrite integral as:

$$\int \sin u \, du$$

$$= -\cos u + C \quad \text{Rewrite solution in terms of } x$$
$$= -\cos(2x) + C$$

Another example:

$$\int \sqrt{\underline{4x^5 - 5x}} (\underline{20x^4 - 5}) \underline{dx} \quad u = \underline{4x^5 - 5x}$$

Rewrite integral in terms of u

$$\frac{du}{dx} = 20x^4 - 5$$

$$du = (20x^4 - 5) dx$$

$$\int u^{\frac{1}{2}} du = \frac{2}{3} u^{\frac{3}{2}} + C$$

Rewrite your solution in terms of x :

$$\int \sqrt{4x^5 - 5x} (20x^4 - 5) dx = \frac{2}{3} (4x^5 - 5x)^{\frac{3}{2}} + C$$

In general, integration by u-sub looks like:

$$\int (\text{composite function})(\text{another function}^*) dx$$

*The other function looks like the derivative of the “inside” of the composite function. Look for the composite function!

Try:

$$\int \frac{10x}{(5x^2 + 7)^3} dx$$

$$u = 5x^2 + 7$$

$$\frac{du}{dx} = 10x$$

$$du = 10x dx$$

$$= \int (5x^2 + 7)^{-3} (10x) dx$$

$$= \int u^{-3} du$$

$$= \frac{u^{-2}}{-2} + C$$

$$= -\frac{1}{2} (5x^2 + 7)^{-2} + C$$

$$\int \tan^2 x \sec^2 x dx$$

$$= \int u^2 du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{\tan^3 x}{3} + C$$

$$u = \tan x$$

$$du = \sec^2 x dx$$

What if we do not quite have du ?

$$\int x^3 (x^4 + 3)^2 dx$$

$$u = x^4 + 3$$

$$\frac{du}{dx} = 4x^3$$

$$du = 4x^3 dx$$

Hey! We only have x^3 , not $4x^3$. No worries, we can “fix” our substitution:

$$\frac{du}{4} = x^3 dx$$

Remember, it’s called u -substitution so YOU need to make all of the substitutions for what you have in the integrand.

$$\int u^2 \frac{du}{4} \quad \text{use CONSTANT MULTIPLE RULE}$$

$$= \frac{1}{4} \int u^2 du$$

$$= \frac{1}{4} \left(\frac{u^3}{3} \right) + C$$

Now rewrite solution in terms of x

$$\int x^3 (x^4 + 3)^2 dx = \frac{1}{12} (x^4 + 3)^3 + C$$

$$\int \cos(3x) dx$$

$$u = 3x$$

$$\frac{du}{dx} = 3$$

$$du = 3 dx$$

$$\frac{du}{3} = dx$$

$$\frac{1}{3} \int \cos u du =$$

$$\frac{1}{3} \sin u + C$$

$$\int \cos(3x) dx = \frac{1}{3} \sin(3x) + C$$

Consider:

$$\int \underline{2x^3} \sqrt{\underline{x^4 + 5}} \underline{dx}$$

$$u = \underline{x^4 + 5}$$

$$du = 4x^3 dx$$

$$\frac{1}{2} du = \underline{2x^3 dx}$$

u-sub work

$$\frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{2} \left(\frac{2}{\frac{3}{2}} u^{\frac{3}{2}} \right) + C$$

$$\int 2x^3 \sqrt{x^4 + 5} dx = \frac{1}{3} (x^4 + 5)^{\frac{3}{2}} + C$$

Try:



$$u = x^2 + 3$$

$$\int x(x^2 + 3)^9 dx$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$= \frac{1}{2} \int u^9 du$$



$$= \frac{1}{2} \left[\frac{u^{10}}{10} \right] + C$$

$$= \frac{1}{20} [x^2 + 3]^{10} + C$$

$$\int \frac{x^5 + x^3}{x^2} dx$$

Do we need u-substitution?

JUST DIVIDE NOPE

$$\int \frac{3x^2}{x^3 + 1} dx$$

Do we need u-substitution?



$$u = x^3 + 1$$

$$du = 3x^2 dx$$

$$\int \sin^2 \theta \cos \theta d\theta$$

Do we need u-substitution?



$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$\int (\sin^2 \theta + \cos^2 \theta) d\theta$ Do we need u-substitution?

$$\sin^2 \theta + \cos^2 \theta = 1$$

No

Here is a different looking one!

$$\int 5x^4 g'(x^5) dx$$

$$u = x^5$$

$$du = 5x^4 dx$$

$$= \int g'(u) du$$

$$= g(u) + C$$

$$= g(x^5) + C$$

PLEASE SHOW ALL STEPS

Homework: page 304 #7, 9, 13, 19, 21, 23