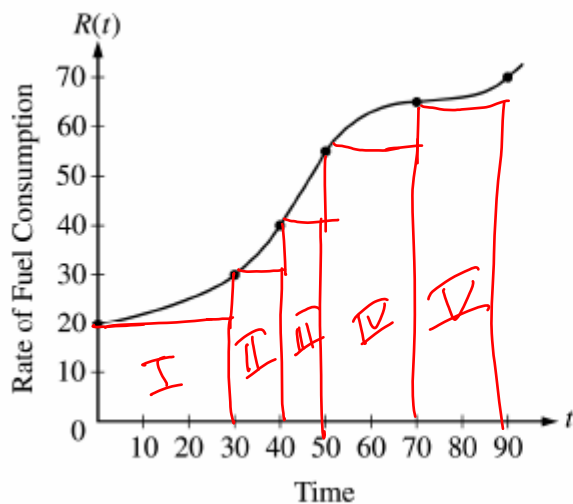


Trapezoid Rule-another geometric approach to definite integrals

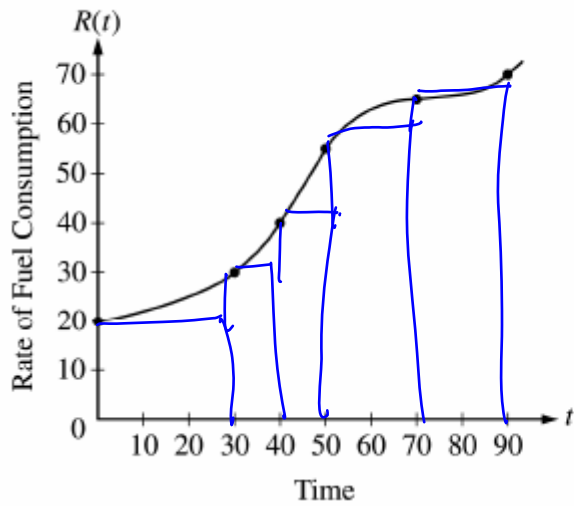
- used when given data or used when we don't know the anti-derivative of the integrand
- similar to Rectangular Approximation Method [RAM] but uses trapezoids not rectangles [hence the name]
- intervals do NOT need to have same width

Let's revisit the airplane problem!
2003 AB 3 [calculator-friendly]



| t (minutes) | $R(t)$ (gallons per minute) |
|------------------|--------------------------------|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.

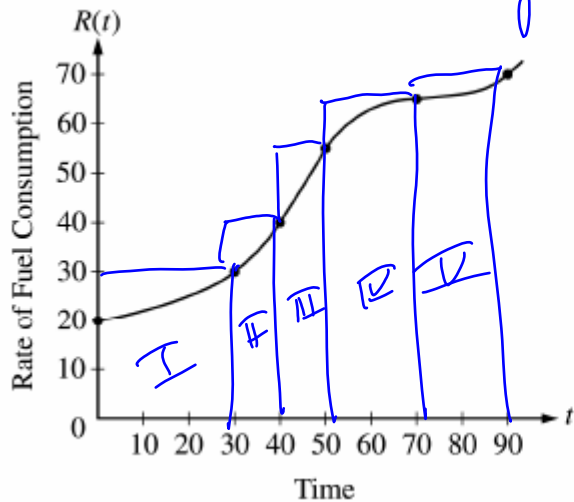


| t (minutes) | $R(t)$ (gallons per minute) |
|------------------|--------------------------------|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

$$\int_0^{90} R(t) dt \approx LRAM$$

$$LRAM = 30 R(0) + 10 R(30) + 10 R(40) + 20 R(50) + 20 R(70)$$

$$= 3700 \text{ gallons}$$

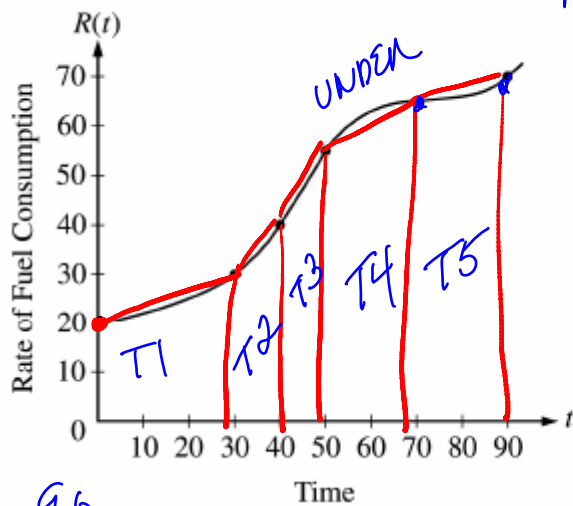


| t (minutes) | $R(t)$ (gallons per minute) |
|------------------|--------------------------------|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

$$RRAM = 30 R(30) + 10 R(40) + 10 R(50)$$

$$+ 20 R(70) + 20 R(90)$$

$$= 4550 \text{ gallons}$$



$$\text{AREA OF TRAPEZOID} = \frac{b_1 + b_2}{2} (h)$$

| t (minutes) | R(t) (gallons per minute) |
|-------------|---------------------------|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

IF C. UP
The
TRAPEZOID
is "OVER"

IF C. DOWN
The TRAP
is "UNDER"

$$\int_0^{90} R(t) dt = \text{TRAP}$$

$$\text{TRAP} = \frac{R(0) + R(30)}{2} (30) + \frac{R(30) + R(40)}{2} (10) + \frac{R(40) + R(50)}{2} (10) + \frac{R(50) + R(70)}{2} (20) + \frac{R(70) + R(90)}{2} (20)$$

$$\text{TRAP} = 4125 \text{ GALLONS}$$

Is LRAM an over- or under-estimate?

Is RRAM an over- or under-estimate?

Is TRAP an over- or under-estimate? *slightly over*

Which is the best estimate? *TRAPEZOID*

Why? *LESS "WASTED" SPACE
FOLLOWS CURVE BETTER
[SECANTS AS OPPOSED TO
HORIZONTAL LINES]*

What do you obtain when you take the average of LRAM and RRAM?

$$\frac{3700 + 4550}{2} = 4125$$

Example 2

Using the table, estimate the total distance traveled from time $t = 0$ to $t = 6$ using LRAM, RRAM, MRAM, and the Trapezoid rule

| | | | | | | | |
|------------------|---|---|---|---|---|---|----|
| Time, t | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Velocity, $v(t)$ | 3 | 4 | 5 | 4 | 7 | 8 | 11 |

Since $v(t) > 0$ on this interval, then the total distance

traveled can be found by $\int_0^6 v(t) dt$

Now let's estimate using LRAM, RRAM, MRAM, and try using trapezoids.

$$\int_0^6 v(t) dt \approx LRAM$$

$$LRAM = 1 [v(0) + v(1) + v(2) + v(3) + v(4) + v(5)]$$

$$= 31$$

$$\int_0^6 v(t) dt \approx RRAM$$

$$RRAM = 1 [v(1) + v(2) + v(3) + v(4) + v(5) + v(6)]$$

$$= 39$$

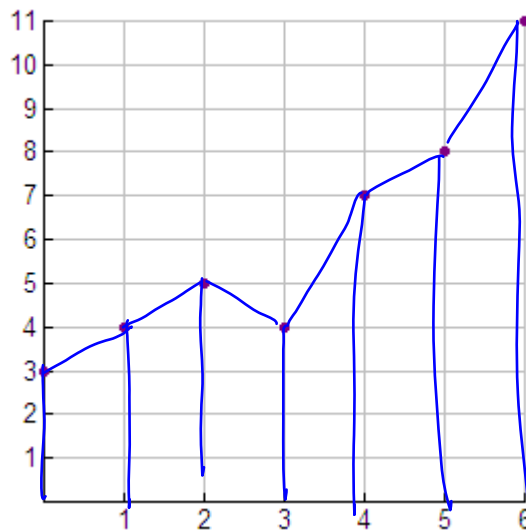
$$\int_0^6 v(t) dt \approx MRAM$$

$$MRAM = 2 [v(1) + v(3) + v(5)]$$

$$= 2 [4 + 4 + 8]$$

$$= 32$$

Let's draw in the trapezoids!



$$\int_0^6 v(t) dt \approx TRAP$$

Let's sum up the areas of each trapezoid and look for a pattern.

$$\frac{1}{2} = \text{GCF}$$

$$\begin{aligned} \text{TRAP} &= (1) \left(\frac{v(0) + v(1)}{2} \right) + (1) \left(\frac{v(1) + v(2)}{2} \right) + (1) \left(\frac{v(2) + v(3)}{2} \right) + \\ &\quad (1) \left(\frac{v(3) + v(4)}{2} \right) + (1) \left(\frac{v(4) + v(5)}{2} \right) + (1) \left(\frac{v(5) + v(6)}{2} \right) \end{aligned}$$

Let's simplify this so that the computation is easier.

$$\begin{aligned} \text{TRAP} &= \frac{1}{2} (v(0) + 2v(1) + 2v(2) + 2v(3) + 2v(4) + 2v(5) + v(6)) \\ &= 35 \end{aligned}$$

Hmm! Isn't 35 the average of LRAM and RRAM? *yes*

There is a formula that you memorize but it is probably just as easy to find the sum of the trapezoid areas. Here is the formula:

Let f be continuous on $[a, b]$. The Trapezoid Rule for

approximating $\int_a^b f(x) dx$ is given by:

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

LH *RH*

Where n is the number of trapezoids, a is the lower bound, and b is the upper bound.

In our example above, $a = 0$, $b = 6$, $n = 6$

$$\frac{6-0}{2(6)} = \frac{1}{2}$$

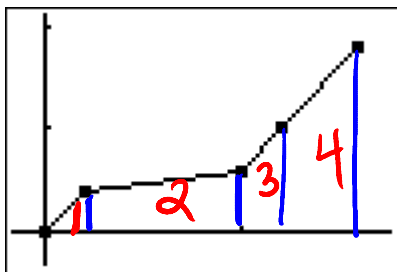
Always be very careful when given data in a table. If you do not have equal subdivisions, then the above formula does not work! If this is the case, then just find the sum of the areas of the trapezoids.

Consider the table of velocity values below and estimate the total distance traveled using the Trapezoid Rule. [Note: $v(t) > 0$ for the interval.]

| | | | | | |
|--------------|---|---|---|---|---|
| t (sec) | 0 | 1 | 5 | 6 | 8 |
| $v(t)$ m/sec | 0 | 2 | 3 | 5 | 9 |

$$TDT = \int_0^8 v(t) dt \approx TRAP$$

We cannot use the formula because we have unequal subdivisions.



$$TRAP = \overset{h}{(1)} \overset{v(0)+v(1)}{\left(\frac{0+2}{2}\right)} + \overset{v(1)+v(5)}{(4)} \left(\frac{2+3}{2}\right) + \overset{v(5)+v(6)}{(1)} \left(\frac{3+5}{2}\right) + \overset{v(6)+v(8)}{(2)} \left(\frac{5+9}{2}\right)$$

$$TRAP = 1 + 10 + 4 + 14$$

$$TRAP = 29m$$

Try using the Trapezoid Rule for #48 on page 315

| | | | | | | | | | |
|---|------|------|------|------|------|------|------|------|------|
| x | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
| y | 4.32 | 4.36 | 4.58 | 5.79 | 6.14 | 7.25 | 7.64 | 8.08 | 8.14 |

Use the Trapezoid Rule to estimate $\int_0^2 f(x) dx$

$$\int_0^2 f(x) dx \approx TRAP$$

$$TRAP = \frac{2-0}{2(8)} \left[y(0) + 2y(0.25) + 2y(0.5) + 2y(0.75) + 2y(1) + 2y(1.25) + 2y(1.5) + 2y(1.75) + y(2) \right]$$

$$TRAP = 12.518$$

Let's re-visit the Heated Wire Problem [2005AB3]
[From our Chapter Four Handout]

| | | | | | |
|--|-----|----|----|----|----|
| Distance x (cm) | 0 | 1 | 5 | 6 | 8 |
| Temperature $T(x)$ ($^{\circ}\text{C}$) | 100 | 93 | 70 | 62 | 55 |

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

$$\frac{1}{8-0} \int_0^8 T(x) dx$$

$$\int_0^8 T(x) dx \approx \text{TRAP}$$

$$\text{TRAP} = (1) \frac{T(0)+T(1)}{2} + (4) \frac{T(1)+T(5)}{2} + (1) \frac{T(5)+T(6)}{2}$$

$$+ (2) \frac{T(6)+T(8)}{2}$$

$$\text{TRAP} = 605.5$$

$$\text{AV TEMP} = \frac{605.5}{8} = 75.6875^{\circ}\text{C}$$

Homework: page 315 #51, 52 AND page 320 10d AND 1999AB3 from the Applications of Integration handout but use the Trapezoid Rule rather than MRAM for part (a)
As always, show all steps