Chapter 5 Review [Some problems to ponder]

Non-calculator problems:

If $f(x) = \tan\left(e^{2x}\right)$, then $f'(x) =$

If $F(x) = \int_{e^3}^{x} \frac{10}{25 + \ln(2t)}\, dt$, then $F'(e^3) =$

Find the average value of $f(x) = \frac{1}{9 + x^2}$ for the closed interval [0, 3]

$$\int_{e}^{e^3} \frac{dx}{x \ln x}$$

If $f(x) = \tan x + e^{-x^2}$, then $f''(0) =$

If $f(x) = \dfrac{e^{-x^2}}{x^2+1}$, then $f'(1) =$

For $x > 0$, $f'(x) = \dfrac{1 + \ln x}{x^2 + 3}$. Find all intervals for which $f(x)$ is increasing.

Calculator problems

If $f'(x) = \ln\left(5 + e^x\right)$ and $f(0) = 10$, then $f(12) =$

Let $f(x) = 7x^2 - 5x + 3$ and let g be the inverse function of f. What is the value of $g'(21)$?

$x > 0$

$21 = 7x^2 - 5x + 3$ $\quad x = 2$

$g'(21) = \dfrac{1}{f'(2)} = \dfrac{1}{23}$

$f'(x) = 14x - 5$

$f'(2) = 28 - 5$

Free Response Problems to consider:

[These are on your Chapter 5 handout]

2003AB 4B [non-calculator]

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$

(a) Find the acceleration of the particle at time $t = 3$

$u = 1-t$

$\frac{du}{dt} = -1$

$a(t) = v'(t) = -e^{1-t}$

$a(3) = -e^{1-3} = -e^{-2}$

(by TI) $a(3) = v'(3) \simeq -.135$

(b) Is the speed of the particle increasing at time $t = 3$? Justify your answer [using Calculus!]

$a(3) < 0$ $\quad$ $v(3) = -1 + e^{1-3}$

$v(3) < 0$

Speed is increasing at $t = 3$ because $a(3)$ and $v(3)$ are both neg.

(c) Find all values of t at which the particle changes direction. Justify your answer [using Calculus!]

$v(t) = 0$

$0 = -1 + e^{1-t}$

$1 = e^{1-t}$

$\ln 1 = \ln e^{1-t}$ "by HAND"

$0 = 1-t$

$1 = t$

on $(0, 1)$ $v(t) > 0$

on $(1, \infty)$ $v(t) < 0$

at $t = 1$ $v(t)$ changes from positive to negative values. $\therefore$ the particle changes direction at $t = 1$.

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

$$TDT = \int_0^3 |v(t)|\, dt$$

positive part of $v(t)$ negative part of $v(t)$

$$= \int_0^1 v(t)\,dt - \int_1^3 v(t)\,dt$$

$$= \left[-t - e^{1-t} \Big|_0^1\right] - \left[-t - e^{1-t} \Big|_1^3\right]$$

$$= e + e^{-2} - 1$$

$$\approx 1.854$$

2007 AB 6 [non-calculator]

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant $f(x) = kx^{\frac{1}{2}} - \ln x$

(a) Find $f'(x)$ and $f''(x)$

$$f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$$

$$f''(x) = -\frac{1}{4}k x^{-\frac{3}{2}} + x^{-2}$$

(b) For what value of the constant k does f have a critical point at $x=1$

$f'(1)=0$

$0=\frac{k}{2\sqrt{1}}-\frac{1}{1}$ Hence, $k=2$

For this value of k, determine whether f has a relative maximum, relative minimum, or neither at $x=1$. Justify your answer.

$f''(x)=-\frac{1}{2}x^{-3/2}+x^{-2}$

$f''(1)>0$

By Second Derivative Test, f has a relative minimum at $x=1$

OR $(0,1)$ $f'<0$ $(1,\infty)$ $f'>0$

at $x=1$ $f'(x)$ changes from neg to positive values, hence f has a rel min at $x=1$

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

$f(x)=0$ $f''(x)=0$

$$0 = k\sqrt{x} - \ln x$$
$$\ln x = k\sqrt{x}$$
$$\frac{\ln x}{\sqrt{x}} = k$$

$$0 = -\frac{1}{4}kx^{-\frac{3}{2}} + x^{-2}$$
$$\frac{1}{4}kx^{-\frac{3}{2}} = x^{-2}$$
$$k = \frac{4x^{-2}}{x^{-3/2}}$$
$$k = \frac{4}{\sqrt{x}}$$

$$\frac{\ln x}{\cancel{\sqrt{x}}} = \frac{4}{\cancel{\sqrt{x}}}$$
$$\ln x = 4$$
$$x = e^4$$
$$k = \frac{4}{\sqrt{e^4}} = \frac{4}{e^2}$$