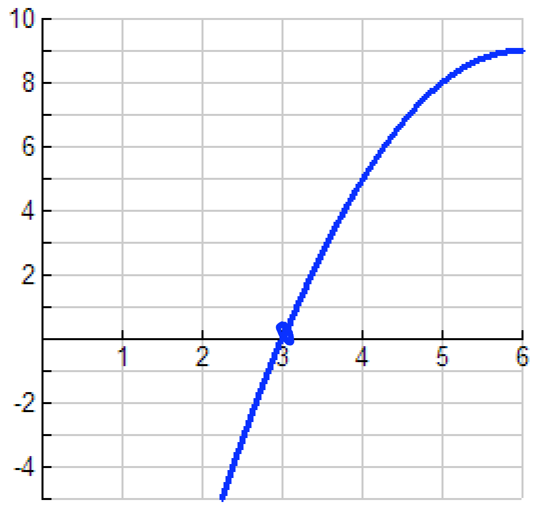
My Chapter Three AP Problems
2009ch3ap.doc

1. The graph of a twice-differential function, f , is shown below. Compare the values of $f(3)$, $f\,'(3)$, $f\,''(3)$



$f(3) = 0$

$f'(3) > 0$
because f increasing

$f''(3) < 0$
because f is concave down

$f''(3) < f(3) < f'(3)$

2. **2001 AB4 [non-calculator]**

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$. $h'(x) = x - 2x^{-1}$ $x \neq 0$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

Do (c) first because it is easy

Point: $(4, -3)$

need slope   find $h'(4)$

$h'(4) = \frac{7}{2}$

equation of tangent line at $x = 4$

$y + 3 = \frac{7}{2}(x - 4)$

(a) horizontal tangent if $h'(x) = 0$

$0 = \dfrac{x^2 - 2}{x}$ if $x = \pm\sqrt{2}$ $x^2 - 2 = 0$

$h''(x) = 1 + \dfrac{2}{x^2}$

use Second Derivative Test $h''(\sqrt{2}) > 0$ $h''(-\sqrt{2}) > 0$

By the Second Der Test $h(x)$ has rel min at $x = \pm\sqrt{2}$

(b) $h''(x) = 1 + \dfrac{2}{x^2}$ $x \neq 0$ $h'' > 0$ for $x \neq 0$

h is concave up for $x \neq 0$ because $h'' > 0$ for $x \neq 0$

3. 2001 AB5 [non-calculator]

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

$f'(-1) = 0$

make sure that f' changes from neg to pos

$f''(-2) = 0$

make sure that f'' changes from either pos to neg or neg to pos

$$f'(x) = 12x^2 + 2ax + b$$

$$f''(x) = 24x + 2a$$

we know $f''(-2) = 0$

$$0 = 24(-2) + 2a$$

$a = 24$

$$f'(x) = 12x^2 + 48x + b$$

we know $f'(-1) = 0$

$$0 = 12 - 48 + b$$

so $b = 36$

$$f(x) = 4x^3 + 24x^2 + 36x + k$$

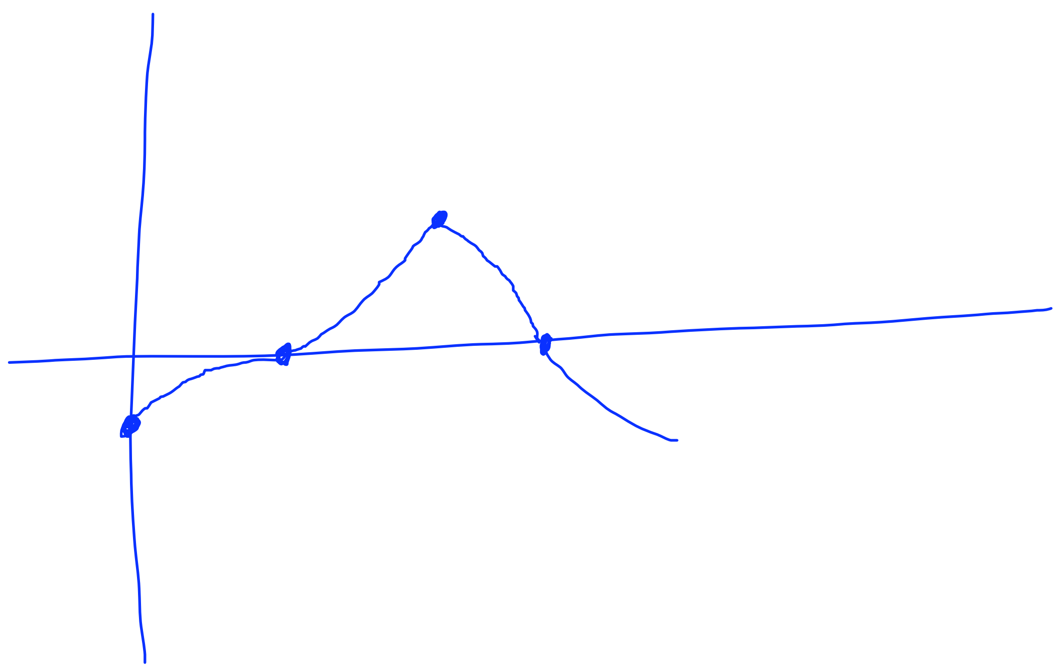
4. 2005 AB 4 [non-calculator]

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

at $x=2$ f' changes from positive to negative values
Hence f has a rel maximum at $x=2$ since $f(2)=2$ our rel max is 2.



Homework for 2 November 2009
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WRITE DOWN
The PROBLEM