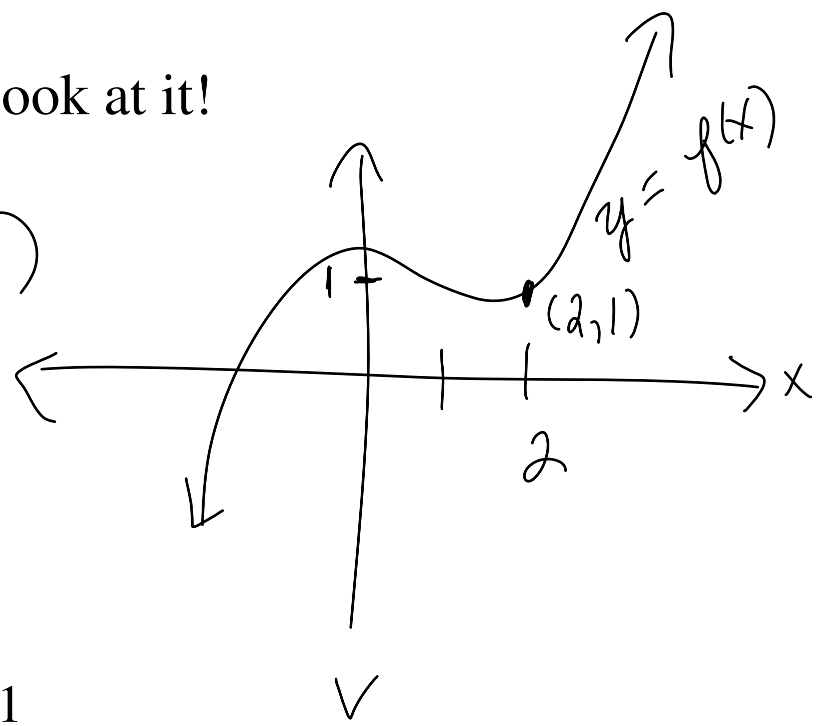
Limit Review

The limit of a function as *x* approaches some given number is the value of the function as the function approaches that given number. In simpler words, the limit of a function is the expected *y*-value of the function as you get very, very close to the given *x*-value.

Finding limits

If given a graph, then look at it!

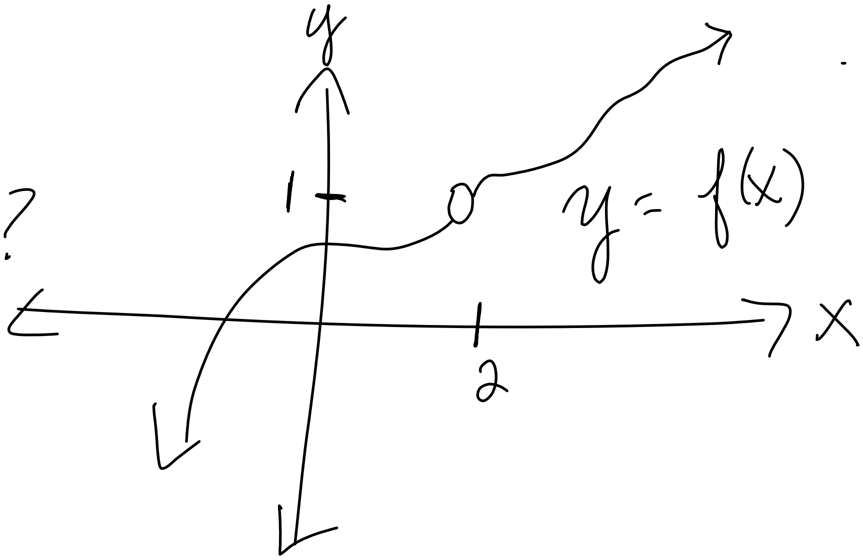
$$\lim_{x \to 2} f(x)$$



It looks like $\lim_{x \to 2} f(x) = 1$

But what if the graph has a "hole" in it? Does the limit still exist?

$$\lim_{x \to 2} f(x) = ?$$

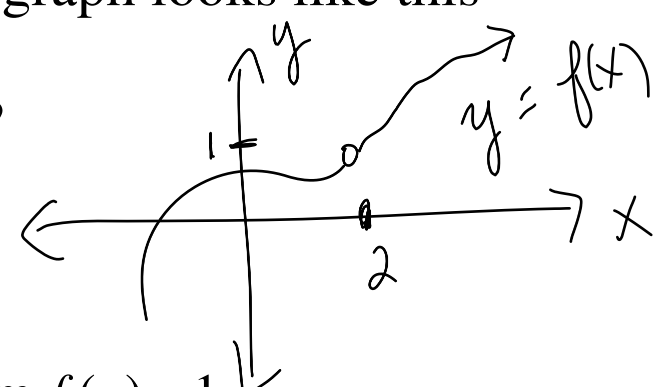


It still looks like $\lim_{x\to 2} f(x) = 1$

As x approaches 2 from the right and the left, the y-value approaches 1. So even though there is a removable discontinuity at x=2, there is still a limit.

Wait! What if the graph looks like this

$$\lim_{x \to 2} f(x) = ?$$



It still looks like $\lim_{x\to 2} f(x) = 1$

Same reason as above!

Finding limits

What if no graph is given? *First, try substituting*.

$$\lim_{x \to 2}(3x-5) = ?$$

Well, $3(2)-5=1$ So, $\lim_{x \to 2}(3x-5)=1$

Does substitution always work? Are you kidding me?!

What happens if we try substitution for this limit:

$\lim_{x \to 5} \frac{x-5}{x^2-25}$ If we try letting x = 5, then we would get $\frac{0}{0}$ which is undefined.

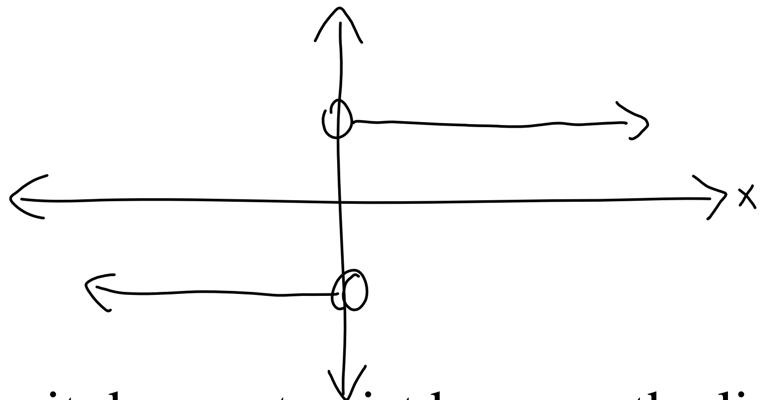
If we get $\frac{0}{0}$, then we could *try simplifying* to see if that helps.

$$\lim_{x \to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x \to 5} \frac{1}{x+5} = \frac{1}{10}$$

Problem solved and we found our limit. Once again, just because a function is undefined at a certain value of x, that does not necessarily mean that the function has no limit at that value of x.

But what if we can't simplify? Then pick up your pencil and graph the function!

$$\lim_{x \to 0} \frac{x}{|x|} = ?$$



Looks like the limit does not exist because the limit as x approaches 0 from the right does not equal the limit as x approaches 0 from the left. Bummer!

Limits involving infinity

What if $\lim_{x \to c} f(x) = \pm\infty$? Then $x = c$ is a vertical asymptote of the function. [This is the Calculus definition of a vertical asymptote and we are in Calculus class.] How can we "spot" a vertical asymptote? Most of the time, when you substitute the value of x into the function, the result will be some number divided by zero.

$$\lim_{x \to 0} \frac{1}{x^2} = ?$$

$$\lim_{x \to \frac{\pi}{2}} \tan x = ?$$

$$\lim_{x \to 3} \frac{(x-2)(x+5)}{(x-2)(x-3)} = ?$$

[The following material is for a later chapter.]
If $\lim_{x \to \pm\infty} f(x) = L$, then $y = L$ is a horizontal asymptote of the function. We called this "end behavior".

$$\lim_{x \to \infty} \frac{2x^2 - 3x + 5}{15 + 3x - x^2} = ?$$

$$\lim_{x \to -\infty} \frac{15 + 3x - x^2}{2x^2 - 3x + 5} = ?$$

$$\lim_{x \to \infty} \frac{2x^3 - 3x + 15}{15 + 3x - x^2} = ?$$

$$\lim_{x \to -\infty} \frac{15 + 3x - x^2}{2x^3 - 3x + 15} = ?$$

Don't forget the properties of limits which can be found in your textbook or in your notes.

What do we use limits for?
Lots of stuff but so far we have used limits to prove continuity.

In this class, we must prove continuity by using Calculus. In this case, that means that we have to use the three-part definition of continuity.

A function f is continuous at $x = c$ if:

1. $f(c)$ exists [i.e., the function is defined at $x = c$]
2. $\lim_{x \to c} f(x)$ exists
3. 1 = 2 from above

Example:

Let $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$

Is this function continuous at $x = 1$?

Another example:

Calculate the values of *a* and *b* that make *g(x)* continuous for all real numbers.

$$g(x) = \begin{cases} x^2, & x < -4 \\ ax + b, & -4 \le x < 5 \\ \sqrt{x + 31}, & x \ge 5 \end{cases}$$

Last, but not least, understand what the *Intermediate Value Theorem* means in terms of continuous functions.

Let $f(x)$ be a continuous function on the interval $-2 \le x \le 2$ with selected values given in the table below.

x	-2	-1	0	1	2
$f(x)$	-5	2	1	1.2	1.3

Show that $f(x)$ must have at least one zero on the interval $-2 \le x \le 2$. [Use Calculus!]

Which of the following statement(s) must be true about $f(x)$:

x	-2	-1	0	1	2
$f(x)$	-5	2	1	1.2	1.3

(a) $\lim_{x \to 0} f(x) = 1$

(b) $\lim_{x \to 2^-} f(x) = 1.3$

Try to draw 3 different graphs that fit the values given in the table for $f(x)$. [Make sure that your graphs are continuous]

Types of Discontinuities:

Removable

Jump

Infinite

Oscillating

Some more practice problems:

1. Draw a graph with the following seven attributes:

I. $f(-3) = 3$

II. $\lim_{x \to -3^-} f(x) = 2$

III. $\lim_{x \to -3^+} f(x) = 4$

IV. $\lim_{x \to 3^-} f(x) = -\infty$

V. $\lim_{x \to 3^+} f(x) = \infty$

VI. $f(0) = 1$

VII. $\lim_{x \to 0} f(x) = 1$

2. What is $\lim_{x \to \infty} \dfrac{x^2 - 9}{3 + x - 9x^2}$?

(A) -3 (B) $-\dfrac{1}{9}$ (C) $\dfrac{1}{3}$

(D) 1 (E) The limit does not exist

3. What is $\lim_{x \to 3} \frac{(3 - x^2)}{(x - 3)}$?

(A) -2

(B) -1

(C) 0

(D) 1

(E) The limit does not exist

4. $\lim_{x \to 1} \frac{x}{\ln x}$ is

(A) 0

(B) 1

(C) e

(D) $\frac{1}{e}$

(E) nonexistent

5. A function $f(x)$ has a vertical asymptote at $x = 2$. The graph of $f(x)$ is increasing for all $x \neq 2$. Which of the following statements are true?

I. $\lim_{x \to 2} f(x) = +\infty$

II. $\lim_{x \to 2^+} f(x) = +\infty$

III. $\lim_{x \to 2^-} f(x) = +\infty$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III