Higher Order Derivatives

From Physics:

$s(t)$ position function [sometimes called $x(t)$]

$s'(t) = v(t)$ velocity function

$s''(t) = a(t)$ acceleration function or $a(t) = v'(t)$

$s'''(t) = j(t)$ jerk $a'(t) = j(t)$

In general:

y' first derivative

y'' second derivative

The second can also be written as $\boxed{\frac{d^2y}{dx^2}}$

y''' third derivative

$y^{(4)}$ fourth derivative $y^{(4)} = \frac{d^4y}{dx^4}$

$y^{(n)}$ n^{th} derivative

$\frac{dy}{dx}$ y'

$\frac{d}{dx} f(x) = f'(x)$

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Given: $v(t) = \frac{100t}{2t+15}$ in $\frac{ft}{\sec}$

Find: $a(5)$, $a(10)$, $a(20)$

We'll need to find $a(t)$ first.

$a(t) = \frac{d}{dt}(v(t))$

$a(t) = v'(t)$

HIGH

$a(t) = \frac{d}{dt}\left(\frac{100t}{2t+15}\right)$

LOW

What rule will we need? What units will we use?

$a(t) = \frac{(2t+15)(100) - (100t)(2)}{(2t+15)^2}$

$a(t) = \frac{1500}{(2t+15)^2}$

t	$a(t)$ $\frac{ft}{sec^2}$
5	$\frac{1500}{625}$
10	$\frac{1500}{1225}$
20	$\frac{1500}{3025}$

Let $f(x) = 4\cos x$. Find $f''(x)$

$f'(x) = -4\sin x$

$f''(x) = -4\cos x$

$f'''(x) = 4\sin x$

$f^{(4)}(x) = 4\cos x$

Let $g(x) = \csc x$. Find $g''(x)$

$g'(x) = \underline{-\csc x}\ \underline{\cot x}$

$g''(x) = \underbrace{\csc x \cot x}_{\mathrm{I}'}\ \underbrace{\cot x}_{\mathrm{II}} + \underbrace{(-\csc^2 x)}_{\mathrm{II}'}\underbrace{(-\csc x)}_{\mathrm{I}}$

Let $h(x) = \dfrac{2x^3 - 3x + 1}{x}$. Find $h''(x)$

$h(x) = 2x^2 - 3 + x^{-1}$

$h'(x) = 4x - x^{-2}$

$h''(x) = 4 + 2x^{-3}$

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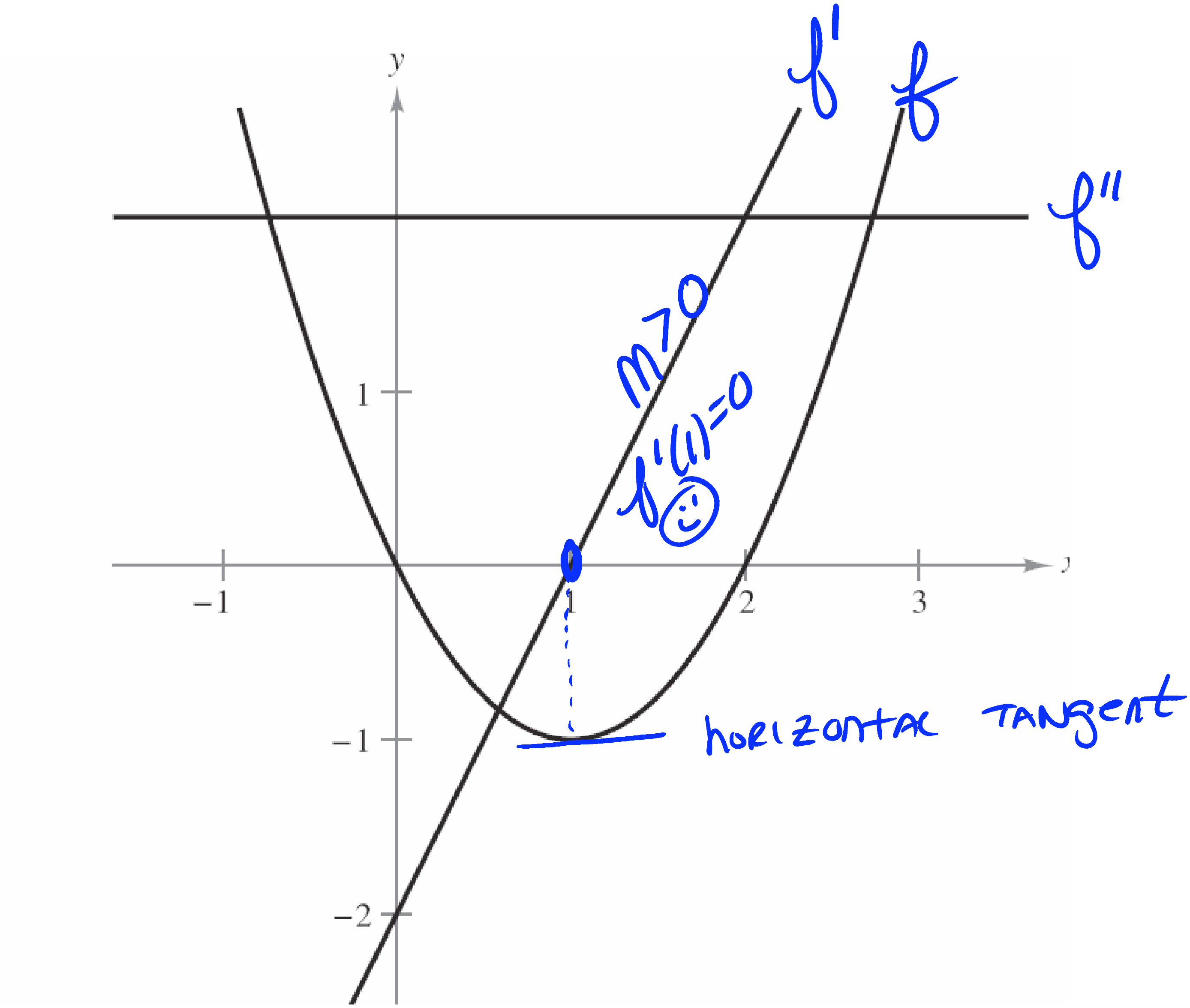


Which is f, f', f''?

See page 128 #105-108

Given: $g(2)=3$, $\underline{g'(2)=-2}$, $h(2)=-1$, $\underline{h'(2)=4}$

Find $f'(2)$

105. $f(x)=2g(x)+h(x)$

$f'(x)=2g'(x)+h'(x)$

$f'(2)=2g'(2)+h'(2)$

$= -4+4 = 0$

106. $f(x)=4-h(x)$

$f'(x)=-h'(x)$

$f'(2)=-h'(2)=-4$

107. $f(x)=\dfrac{g(x)}{h(x)}$

$f'(x)=\dfrac{h(x)g'(x)-g(x)h'(x)}{h^2(x)}$

$f'(2)=\dfrac{h(2)g'(2)-g(2)h'(2)}{[h(2)]^2} = \dfrac{2-12}{(-1)^2}$

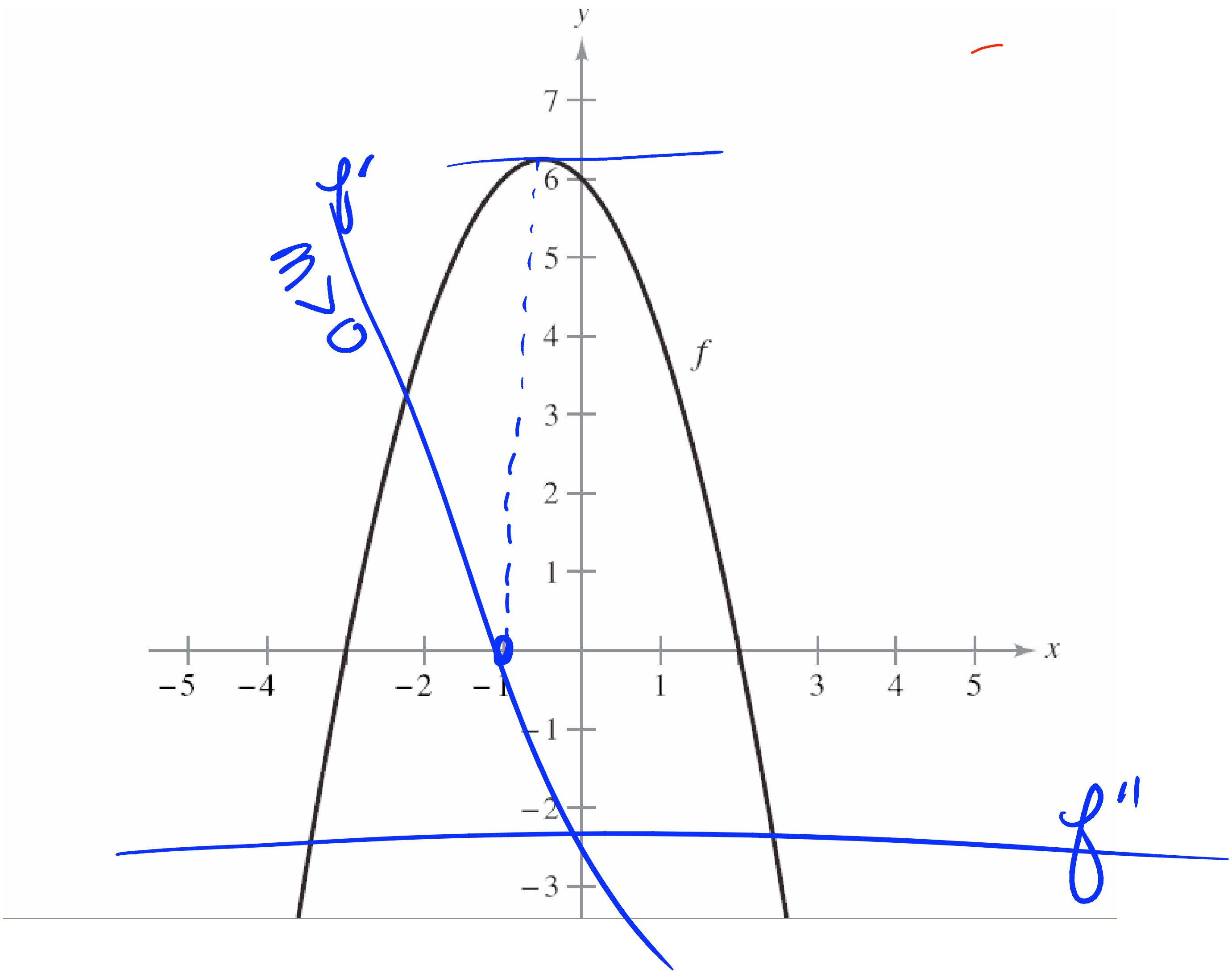
108. $f(x)=\underline{g(x)}\,\underline{h(x)}$

$f'(x)=g'(x)h(x)+h'(x)g(x)$

$f'(2)=g'(2)h(2)+h'(2)g(2)$

$=14$

See page 128 #111 Sketch f' and f''



Homework: page 128 #94-102 evens, plus page 129 #118a