Average Velocity versus Instantaneous Velocity

Given a position function find the average velocity

Average velocity can be found by:

$$av\,vel = m_{\sec} = \frac{\Delta s}{\Delta t} = \frac{change\ in\ position}{change\ in\ time}$$

$s(t) =$ position function [also called $x(t)$]

Let $s(t) = -16t^2 + 50$ where $t =$ time in seconds and $s(t) =$ feet

What is the average velocity on [0, 1]

$$av\,vel = \frac{s(1) - s(0)}{1-0} \qquad \frac{FT}{SEC}$$

$$= \frac{34-50}{1}$$

$$= -16\ \frac{ft}{\sec}$$

Since this is negative, the object is moving either left or in a downwards direction.

Now find the average velocity for the following intervals of time:

[1, 1.5] AND [1, 1.1] [Use your @#%! TI]

$$\text{Av vel} = \frac{s(1.5) - s(1)}{1.5 - 1} = \frac{14 - 34}{.5} = -40 \frac{\text{ft}}{\text{sec}}$$

Av vel

$$\text{Av vel} = \frac{s(1.1) - s(1)}{1.1 - 1} = \frac{30.64 - 34}{.1} = -33.6 \frac{\text{ft}}{\text{sec}}$$

Av vel

Instantaneous velocity $= v(t) = s'(t)$

Speed $= |v(t)|$

Velocity may be positive, negative, or equal to zero

Speed must be ≥ 0

Acceleration $= a(t) = v'(t)$

If $v(t)$ and $a(t)$ have the same sign for some value of t, then the speed is increasing at that value of t ☆☆☆☆

Now, that we know that $v(t) = s'(t)$ we can easily find the instantaneous velocity at some value of t

Let's find $v(1)$ [It should be close to our last average velocity.]

$v(t) = s'(t)$

$v(t) = \frac{d}{dt}\left(-16t^2 + 50\right)$

$v(t) = -32\ t\ \frac{ft}{\text{sec}}$

So, $v(1) = -32\frac{ft}{\text{sec}}$

speed at $t=1$?

$32\ \frac{ft}{sec}$

$v(1) = -32\ \frac{ft}{sec}$

$a(t) = v'(t)$

$a(t) = -32\ \frac{ft}{sec^2}$

$a(1) = -32\ \frac{ft}{sec^2}$

at $t=1$, speed is increasing because $v(1) < 0$ and $a(1) < 0$.

Let's do page 117 #94

A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. What is its velocity after 3 seconds? What is its velocity after falling 108 feet?

Given: $s(t) = -16t^2 + v_o t + s_0$

Where v_0 is the initial velocity and s_0 is the initial position

Hence, for this problem, $s(t) = -16t^2 + -22t + 220$

We need to first find $v(t)$

$v(t) = s'(t)$

$a(t) = v'(t)$
$= -32$

$$v(t) = \frac{d}{dt}\left(-16t^2 - 22t + 220\right)$$

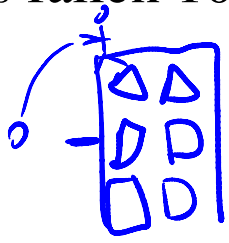
$$v(t) = -32t - 22$$

Its velocity after 3 seconds will be $v(3)$

$$v(3) = -32(3) - 22 = -118\frac{ft}{\text{sec}}$$

at $t = 3$
$v(3) < 0$
$a(3) < 0$
Thus, speed is increasing

Now to find the velocity when the ball has fallen 108ft



If the ball fell 108 feet, then its position would be 220 – 108 = 112 ft

We need to find the **time** when the ball is at 112 ft

Let $s(t) = 112$ and solve for t

$112 = -16t^2 - 22t + 220$

$0 = -16t^2 - 22t + 108$

Hence, $t = 2$ seconds

Now we just need to find $v(2)$

$v(2) = -32(2) - 22 = -86 \frac{ft}{\text{sec}}$

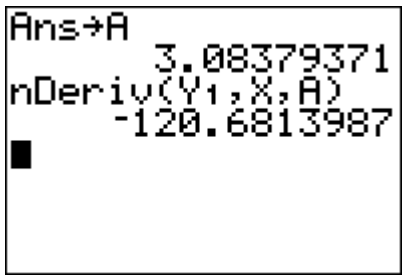
What is the ball's terminal velocity? [What is its velocity when it hits the ground?]

First find when it hits the ground by letting $s(t) = 0$
[Your TI will be handy for this! Pick it up and use it!]

$s(t) = 0$

$0 = -16t^2 - 22t + 220$

$t \approx$ 3.083793 7 sec



Terminal velocity is $v(3.083) =$ $-120.6813587 \frac{ft}{sec}$

Use your @#$#$%^! TI to find the value

During the first 40 seconds of a rocket flight, the rocket is propelled straight up so that in t seconds it reaches a height given by $s(t) = \frac{t^3}{\sqrt{10}}$ feet

(a) What is its position at $t = 40$ seconds?

$s(40) = 20238.577$ ft

(b) What is its average velocity on [0, 40] ?

$\text{AV vel} = \frac{s(40) - s(0)}{40 - 0} = 505.964 \frac{ft}{sec}$

$s(t) = \frac{1}{\sqrt{10}} t^3$

(c) Find its velocity at $t = 40$ seconds

$s'(t) = v(t) = \frac{3}{\sqrt{10}} t^2$

$v(40) = 1517.893 \frac{ft}{sec}$

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Y1(40)
        20238.57703
Ans/40
        505.9644256
nDeriv(Y1,X,40)
        1517.893277
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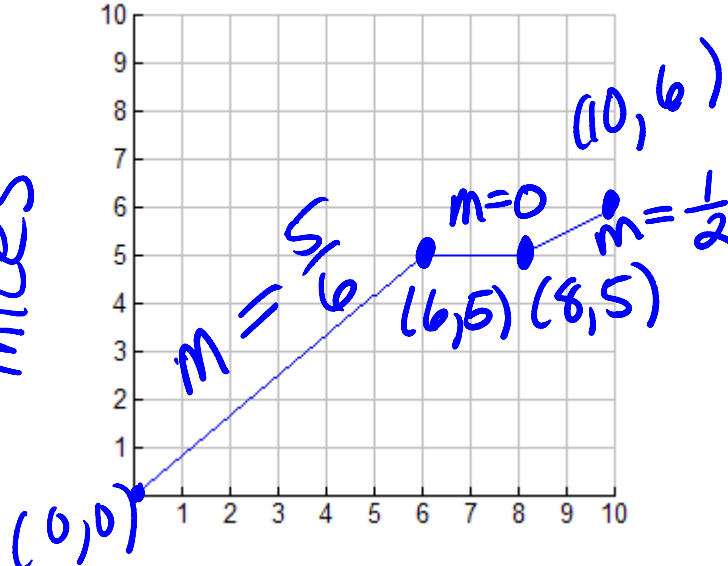
See page 117 #98

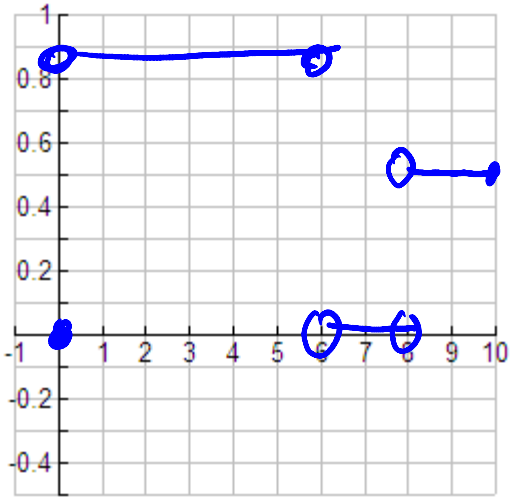
$\frac{6 \text{ miles}}{\frac{1}{6} \text{ hour}} = 36 \text{mph}$



GRAPH OF $s(t)$

$v(6) = s'(6)$ NOT defined

$v(8) = s'(8)$ NOT defined



slopes of line segments of s(t) are the values for v(t)

GRAPH OF v(t)

Homework: pg 117 #93, 95, 97; pg 118 #103, 104