Integrating with e^x

$$\int e^x \, dx = e^x + C$$

Using 2nd FTC

$$\frac{d}{dx}\int_0^x e^t \, dt = e^x$$

Slightly harder:

$$\frac{d}{dx}\int_0^{x^2} e^t \, dt = \left(e^{x^2}\right)\left(\frac{d}{dx} x^2\right)$$

$$= 2xe^{x^2}$$

Another ✵✵✵✵ rule!

$\int e^u \, du = e^u + C$ where u is a differentiable function of x

$$\int 2e^{2x} \, dx$$

Let $u = 2x$

$du = 2dx$

Rewrite as:

$$\int e^u du = e^u + C$$

$$= e^{2x} + C$$

$\int 10e^{10x}\,dx$ Let $u = 10x$

$du = 10\,dx$

$= \int e^u\,du$

$= e^u + C = e^{10x} + C$

Here's a cool one: same $\int \frac{e^{\sin x}}{\sec x}\,dx$

$\int (\cos x)(e^{\sin x})\,dx$ Let $u = \sin x$

$du = \cos x\,dx$

$= \int e^u\,du$

$= e^u + C = e^{\sin x} + C$

Slightly harder:

$\int \frac{e^{-x}}{1+e^{-x}}\,dx$ Let $u = 1+e^{-x}$

$du = -e^{-x}\,dx$

$-du = e^{-x}\,dx$

$= -\int \frac{1}{u}\,du$

$= -\ln|u| + C$

$= -\ln|1+e^{-x}| + C$ $\boxed{♪\ 1+e^{-x} > 0}$

FANCY-PANTS ANSWER $\ln \frac{1}{1+e^{-x}} + C$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

Let's rewrite this as:

$$= \int e^{\sqrt{x}} \left(x^{\frac{-1}{2}} \right) dx$$

Let $u = \sqrt{x} = x^{\frac{1}{2}}$

$du = \frac{1}{2} x^{-\frac{1}{2}} dx$

$2\, du = x^{-\frac{1}{2}}\, dx$

$$= 2\int e^{u}\, du$$

$$= 2e^{u} + C$$

$$= 2e^{\sqrt{x}} + C$$

u-SUB

This one only looks hard: [♪ the division bar!]

$$\int \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}\, dx$$

$u = e^{x} - e^{-x}$

$du = (e^{x} + e^{-x})\, dx$

$$= \int \frac{1}{u}\, du$$

$$= \ln|u| + C$$

$$= \ln|e^{x} - e^{-x}| + C$$

u-SUB

$\int e^x \sqrt{1-e^x} \; dx$ [♪ the grouping symbol]

$= \int e^x (1-e^x)^{\frac{1}{2}} \, dx$

$u = 1 - e^x$

$du = -e^x dx$

$-du = e^x dx$

$= -\int u^{\frac{1}{2}} \, du$

$= -\frac{2}{3} u^{\frac{3}{2}} + C$

$= -\frac{2}{3} (1-e^x)^{\frac{3}{2}} + C$

This one is not as obvious as the other e^u-type integrals:

$\int \frac{5-e^x}{e^{2x}} dx$ *Rewrite as*:

$= \int \frac{5}{e^{2x}} \, dx \; - \; \int \frac{e^x}{e^{2x}} \, dx$ *Rewrite some more*:

$= \int 5e^{-2x} \, dx - \int e^{-x} \; dx$ Now start looking for u

$= -\frac{5}{2} e^{-2x} + e^{-x} + C$

$5 \int e^{-2x} dx$

$u = -2x$

$du = -2dx$

$-\frac{1}{2} du = dx$

$-\frac{5}{2} \int e^u du$

$u = -x$

$du = -dx$

$-du = dx$

$- \int e^u du$

u-SUB

$\int e^{-x}\tan\left(e^{-x}\right)dx$

$u = e^{-x}$

$du = -e^{-x}dx$

$-du = e^{-x}dx$

$= -\int \tan u\, du$

$= \ln|\cos u| + C$

$= \ln|\cos e^{-x}| + C$

$\int \tan u\, du$

$= -\ln|\cos u| + C$

or

$\ln|\sec u| + C$

Definite integrals with e^x, e^u [Same as all the other definite integrals that we have done so far.]

$\int_0^1 e^{-2x}\,dx$

$u = -2x$

$du = -2\,dx$

$-\frac{1}{2}du = dx$

$u(0) = 0$

$u(1) = -2$

$= -\frac{1}{2}\int_0^{-2} e^u\,du$

$= \frac{1}{2}\int_{-2}^{0} e^u\,du$

[NOW WE ARE THROUGH WITH x]

$= \frac{1}{2}\left[e^u\Big|_{-2}^{0}\right] = \frac{1}{2}\left[1 - e^{-2}\right]$

$$\int_0^1 x e^{-x^2}\, dx$$

$u = -x^2$

$du = -2x\, dx$

$-\frac{1}{2} du = x\, dx$

$u(0) = 0$

$u(1) = -1$

$$= -\frac{1}{2}\int_0^{-1} e^u\, du$$

$$= \frac{1}{2}\int_{-1}^{0} e^u\, du$$

$$= \frac{1}{2}\left[e^u \Big|_{-1}^{0}\right] = \frac{1}{2}\left[1 - e^{-1}\right]$$

What we learned today:

$\int e^x\, dx = e^x + C$ **and** $\int e^u\, du = e^u + C$

If it is not e^x, then it is e^u !!!!!

Homework: page 358 # 85, 86, 90, 100, 103, 104