EXTREMA* OF A FUNCTION
*** plural of extreme**

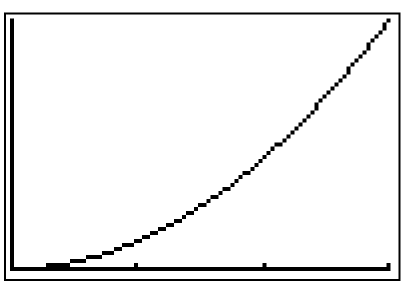
Let f be defined on an interval I containing c

$f(c)$ is the minimum of f on I if $f(c) \leq f(x)$ for all x in I [Translation: "the lowest y-value"]

$f(c)$ is the maximum of f on I if $f(c) \geq f(x)$ for all x in I [Translation: "the greatest y-value"]

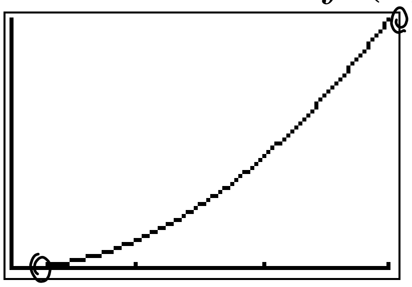
Extreme Value Theorem [existence theorem]
If f is **continuous** on a **closed interval,** $[a,b]$, then f has both a minimum and a maximum on the interval

Consider $f(x) = x^2$ on [0, 3]



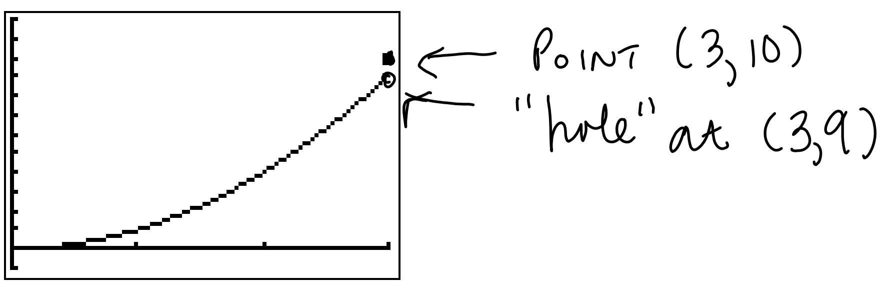
The absolute minimum is 0 at the point (0, 0)
The absolute maximum is 9 at the point (3, 9)

Now consider $f(x) = x^2$ on (0, 3)



Since this is an open interval, the endpoints are not included. There is neither a min nor a max, BUT notice that the **EVT does not apply** because we have an open interval.

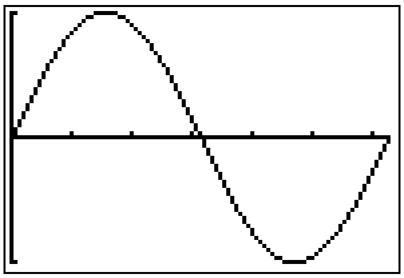
How about $g(x) = \begin{cases} x^2, & 0 \le x < 3 \\ 10, & x = 3 \end{cases}$



EVT does not apply because $g(x)$ is not continuous on [0, 3] but it is defined on [0, 3]

Extrema may occur at endpoints of a closed interval or at an interior point.

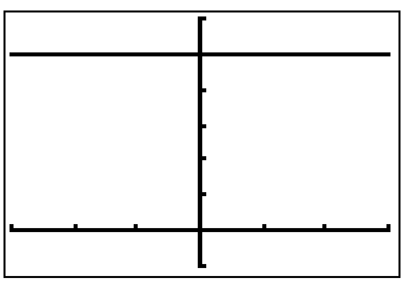
Consider $f(x) = \sin x$ on $[0, 2\pi]$



The absolute minimum value is – 1 at the point $\left(\frac{3\pi}{2}, -1\right)$

The absolute maximum value is 1 at the point $\left(\frac{\pi}{2}, 1\right)$

Consider the function $y = 5$ on $[-3, 3]$



Since the function is continuous on the closed interval, then the EVT must apply. Some people think of this as a weird case of the EVT because the absolute minimum value = absolute maximum value.

Relative [local] versus Absolute [global] extrema

If there is an *open interval* containing c on which $f(c)$ is a maximum, then $f(c)$ is called a relative [local] maximum of f.
Who has the relative maximum height in the last row?

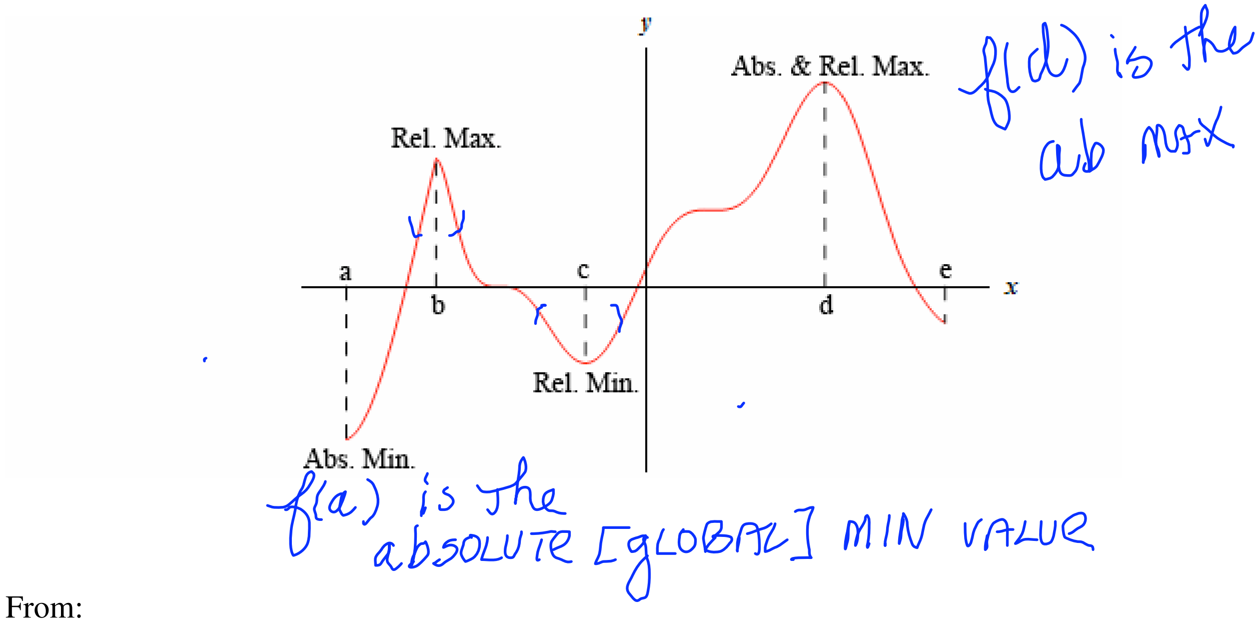
Does this mean that this person has the maximum height in the school?

If there is an ***open interval*** containing c on which $f(c)$ is a minimum, then $f(c)$ is called a relative [local] minimum of f.
Who has the relative minimum age in the middle row?

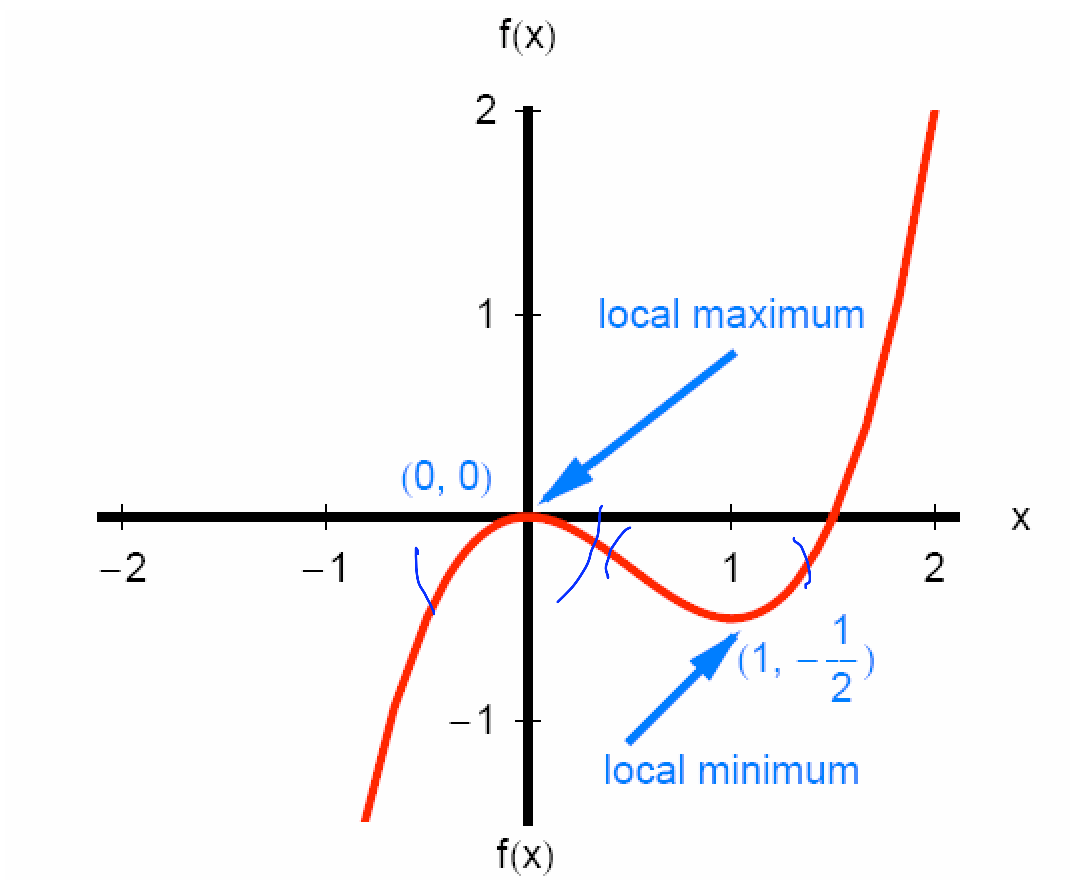
Does this mean that this person has the minimum age in the school?

Let's look at this graphically:

From:
http://tutorial.math.lamar.edu/Classes/CalcI/MinMaxValues.aspx



From:
http://www.frapanthers.com/teachers/zab/CalculusABReview/ReviewUsingDerivatives.pdf



See page 169 # 1 and # 2

#1 A rel min, B ab MAX, E Rel max, F ~~ab~~ rel min, G rel max

#2 A ab min, B rel max, D rel min, E rel MAX,
G rel MIN

Also see page 170 #55 – 58

55.

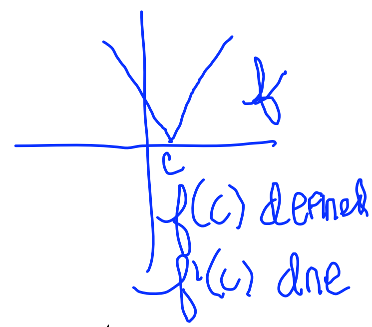
56.

57.

58

Critical number(s) or critical value(s)

Let f be defined at c. If $f'(c)=0$ OR $f'(c)$ is undefined, then c is a critical number of f.



If f has a relative minimum or a relative maximum at $x=c$, then c is a critical number of f. In other words, the critical number(s)/value(s) are worth taking a look at.

Extrema occur at either endpoints of a closed interval or at critical values of f. If given a closed interval, we must consider the endpoint values.

WARNING!!!!
$f'(c) = 0$ or undefined will give us the *possible* extrema of f. Do not assume that every critical value is an extreme value.

ANOTHER WARNING
You must use calculus to justify an extreme value. You may NOT just draw a graph and label extrema.

How about another song from ***Calculus the Musical!***

Maxima and minima

MADE UP WORD

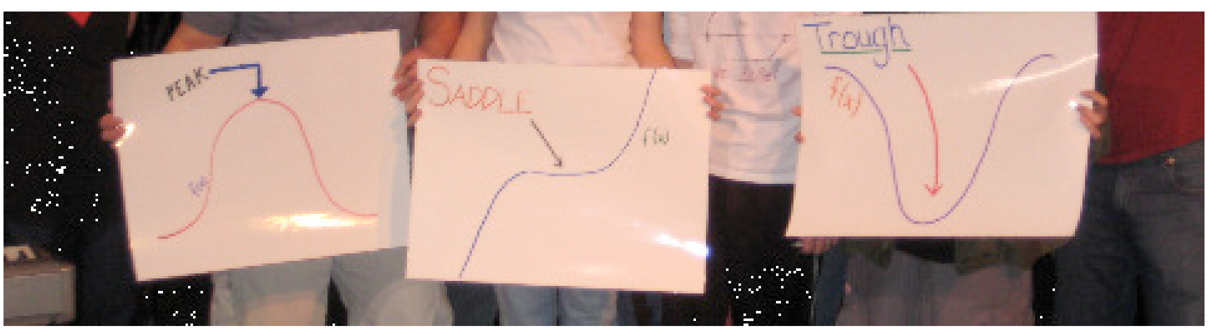
For maxima and minima just take "derivitinima"
Happiness, now just assess the zero, zero, zero, zero!
Don't forget you must inspect the endpoints as they are suspect!
Find the values of our function, look for high and low!
Local maxima are on an interval,
Local minima are on an interval,
Global maxima aren't on an interval
Global minima aren't on an interval! – terval! –terval! –terval!

And
Saddle, Peak and Trough and saddle, peak, and trough and saddle, peak, and trough and peak and trough and

peak and trough and saddle, peak and trough and saddle, peak and trough and saddle, saddle, peak and trough and peak and trough.

Now maxima and minima are also called the extremum,
Sometimes they can be absolute as long as there's no greater, lesser,
Relative implies a region that the extrema is in.
Don't confuse a saddle point! [3 times]
With an extrema!

Saddle, Peak and Trough and saddle, peak, and trough and saddle, peak, and trough and peak and trough and peak and trough and saddle, peak and trough and saddle, peak and trough and saddle, saddle, peak and trough and trough.



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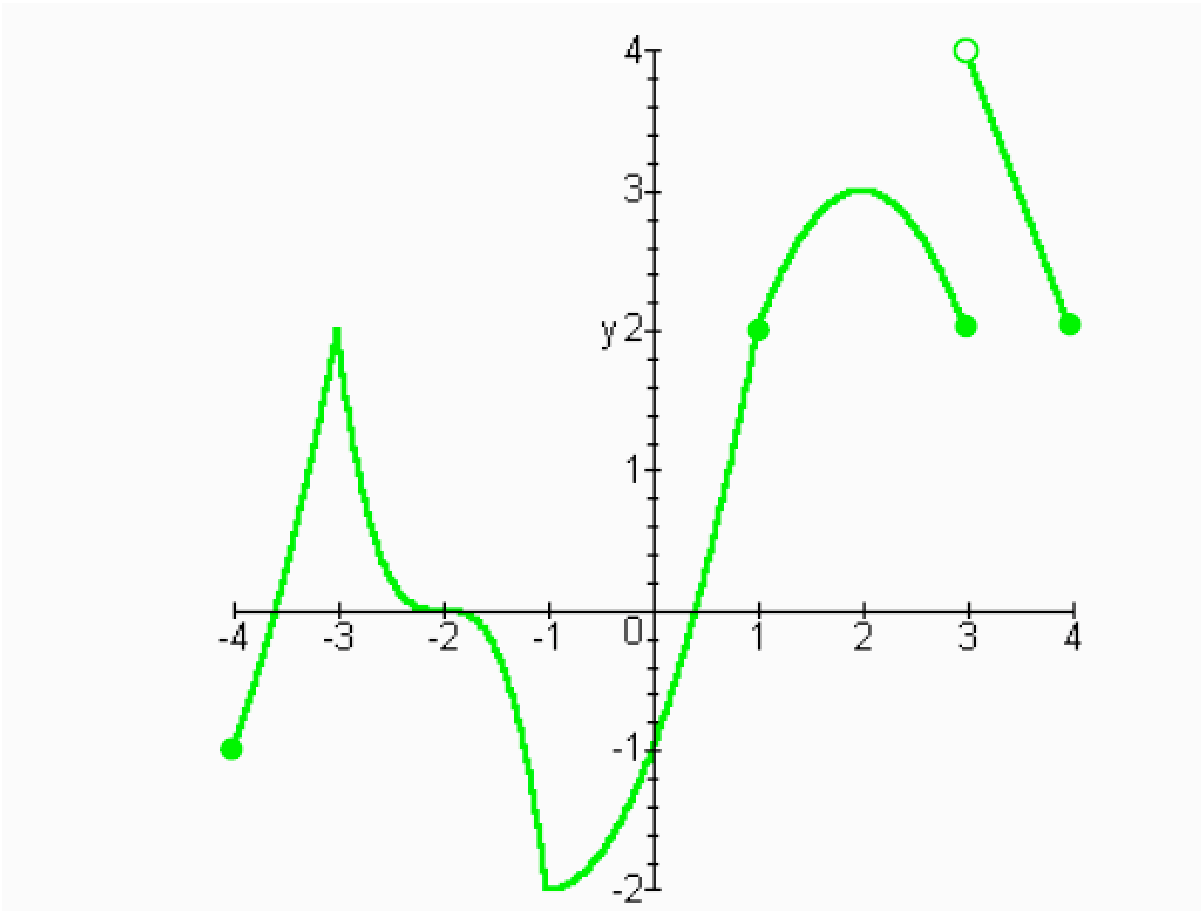
Warm-up for Monday [graph from HOUSTONACT]



The graph above is the graph of f. Does it have any critical values? [In other words, at what values of x is $f'(x)$ either equal to zero or is undefined.]

Now let's do some more problems but without calculators:

Let $f(x) = x^2$ on $[-3,3]$ This is a continuous function on this interval so EVT will apply.

Step one: find $f'(x)$

$$f'(x) = 2x$$

Step two: find any critical values – in other words, find where the derivative equals zero or is undefined.

$$0 = 2x$$

Critical value: $x = 0$

Step three: evaluate the function at the endpoints and the critical value(s)

$f(0) = 0$ absolute minimum is zero

$f(-3) = 9 = f(3)$ absolute maximum is nine

Let $g(x) = 2x - 1$ on [0, 3] This is a continuous function on this interval so EVT will apply.

$g'(x) = 2$ So there are NO critical values so we only have to check the endpoints

$g(0) = -1$ absolute minimum

$g(3) = 5$ absolute maximum

See guidelines for finding extrema on a closed interval on page 167

Let $h(x) = \sqrt[3]{x}$ on $[-1,1]$ This is a continuous function on this interval so EVT will apply.

$$h'(x) = \frac{1}{3x^{\frac{2}{3}}} \text{ or } \frac{1}{3}x^{\frac{-2}{3}}$$

Critical value at $x = 0$ because $h'(0)$ is undefined.

$h(-1) = -1$ absolute minimum

$h(0) = 0$

$h(1) = 1$ absolute maximum

♪ The critical value did NOT give us an extreme value. But we must always check. Constant vigilance!

Try the following:

(1) $h(x) = |x|$ on $[-1, 2]$

Be careful! You should rewrite this as a piecewise function before you look for critical values.

(2) $g(x) = x^{\frac{2}{3}}$ on $[-1, 1]$

(3) $f(x) = \dfrac{x}{x-2}$ on $[3, 5]$

(4) $h(x) = \sec x$ on $\left[\dfrac{-\pi}{6}, \dfrac{\pi}{6}\right]$

Homework:
Page 169 #13-18 all
Remember: A critical value is where the first derivative is either ***undefined*** [but the function is defined] or the first derivative equals ***zero***.