More trigonometric derivatives

There are six trigonometric functions. The two "popular" ones are $\sin x$ and $\cos x$. The other four "less popular" functions can be described in terms of $\sin x$ and/or $\cos x$. So far we know:

$\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$

Based on all of our handy-dandy Differentiation Rules, what would $\frac{d}{dx}(\tan x)$ be? Let's rewrite first!

$$\frac{d}{dx}(\tan x)$$

$$= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

Now we can just use the Quotient Rule.

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

low d high − high d low

$$=\frac{(\cos x)(\cos x)-(\sin x)(-\sin x)}{(\cos x)^2}$$

(low)²

$$=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$

$$=\frac{1}{\cos^2 x}$$

$$=\sec^2 x$$

$\frac{d}{dx}\tan x = \sec^2 x$

Unless you want to do this process every time you need $\frac{d}{dx}(\tan x)$, then you might just want to memorize it!

Using the same method, find $\frac{d}{dx}(\sec x)$, $\frac{d}{dx}(\csc x)$, and $\frac{d}{dx}(\cot x)$

$$=\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

$$=\frac{(\sin x)(-\sin x)-(\cos x)(\cos x)}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x$$

$$\frac{d}{dx}\sec x$$

$$= \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \sec x \tan x$$

$$\frac{d}{dx}\csc x$$

$$= \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= \frac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x}$$

$$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -\csc x \cot x$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Now let's use our new found knowledge!

$$y = x + \cot x$$

$$y' = 1 - \csc^2 x$$

$$g(x) = \frac{\sec x}{x}$$

$$g'(x) = \frac{(x)(\sec x \tan x) - (\sec x)(1)}{x^2}$$

$$= \frac{\sec x\,(x \tan x - 1)}{x^2}$$

$$h(x)=\frac{1}{x}-10\csc x$$

$$h(x)=x^{-1}-10\csc x$$

$$h'(x)=-x^{-2}+10\csc x\cot x$$

$$f(\theta)=\underline{\sin\theta}\,\underline{\cos\theta}$$

$$f'(\theta)=\underset{\mathrm{I}'}{\cos\theta}\,\underset{\mathrm{II}}{\cos\theta}+\underset{\mathrm{II}'}{(-\sin\theta)}\,\underset{\mathrm{I}}{(\sin\theta)}$$

$$f'(\theta)=\cos^2\theta-\sin^2\theta$$

$$h(\theta)=\underline{5\theta}\,\underline{\sec\theta}+\underline{\theta}\,\underline{\tan\theta}$$

$$h'(\theta)=\underset{\mathrm{I}'}{5}\,\underset{\mathrm{II}}{\sec\theta}+\underset{\mathrm{II}'}{5\theta}\,\underset{\mathrm{I}}{\sec\theta\tan\theta}$$

$$+\underset{\mathrm{I}'}{\tan}\,\underset{\mathrm{II}}{\theta}+\theta\,\underset{\mathrm{II}'}{\sec^2}\underset{\mathrm{I}}{\theta}$$

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