More of the Product Rule and Quotient Rule

THE DERIVATIVE SONG

[Sung to the tune of Happy Birthday to You]
One prime two plus two prime one
One prime two plus two prime one
I just did the product rule,
That means I am Cool!

The quotient rule I need to know
Low d high less high d low
Draw a line then there below
Put the square of the low

Let's practice:

page 126 #7, 9, 11, 20, 22, 25, 29, 33

7 $\frac{x}{x^2+1} = f(x)$

WHEN (WHERE) WILL $f(x)$ HAVE HOR TAN

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{1-x^2}{(x^2+1)^2}$$

$x = 1, -1$

$f'(-1) = f'(1) = 0$
HOR TANS

(9) $\dfrac{\sqrt[3]{x}}{x^3+1} = h(x)$

$$h'(x) = \frac{(x^3+1)\left(\frac{1}{3}x^{-\frac{2}{3}}\right) - (\sqrt[3]{x})(3x^2)}{(x^3+1)^2}$$

(11) $g(x) = \dfrac{(\sin x)}{x^2}$

$$g(x) = (\sin x)(x^{-2})$$

$$g'(x) = (\cos x)(x^{-2}) + (-2x^{-3})(\sin x)$$

(20) $y = \dfrac{5x^2-3}{4}$

$y = \frac{5}{4}x^2 - \frac{3}{4}$ SIMPLIFY TO MAKE "SIMPLER"

$$y' = \frac{5}{2}x$$

(22) $y = \dfrac{4}{5x^2}$ REWRITE TO MAKE SIMPLER

$$y = \frac{4}{5}x^{-2}$$

$$y' = -\frac{8}{5}x^{-3}$$

(25) $$f(x) = \frac{3-2x-x^2}{x^2-1}$$

$$f'(x) = \frac{(x^2-1)(-2-2x)-(3-2x-x^2)(2x)}{(x^2-1)^2}$$

(29) $$f(x) = \frac{2x+5}{\sqrt{x}}$$

$$f(x) = 2x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}$$

$$f'(x) = x^{-\frac{1}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$$

OR $$f'(x) = \frac{(\sqrt{x})(2)-(2x+5)(\frac{1}{2}x^{-\frac{1}{2}})}{x}$$

$$f(x) = \frac{(2x-5)}{2\sqrt{x}}\frac{(\sqrt{x})}{}$$

$$f'(x) = 2\sqrt{x} + \frac{1}{2}(x^{-\frac{1}{2}})(2x-5)$$

Now let's do some "weird" problems from our handout

11. Let $f(7)=0$, $f'(7)=14$, $g(7)=1$ and $g'(7)=\frac{1}{7}$.

Find $h'(7)$ if $h(x)=\frac{f(x)}{g(x)}$

$$h'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$

$$h'(7)=\frac{g(7)f'(7)-f(7)g'(7)}{(g(7))^2}$$

$$=\frac{(1)(14)-0}{1^2}$$

$$\boxed{=14}$$

Now let's do a and b

12. From: http://www.wildstrom.com

Use the following table of values to compute the requested derivatives.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	–3	4	5	–1
2	5	1	3	7
3	–2	2	1	8
4	6	5	2	–3
5	9	3	4	–4

a. $H(x)=5f(x)-3g(x)$. Find $H'(1)$

$$H'(x)=5f'(x)-3g'(x)$$

$$H'(1)=5f'(1)-3g'(1)$$

$$H'(1)=5(5)-3(-1)=28$$

b. $H(x)=\frac{f(x)}{g(x)}$. Find $H'(4)$

$$H'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{g^2(x)}$$

$$H'(4)=\frac{(5)(2)-(6)(-3)}{(5)^2}=\frac{28}{25}$$

c. $H(x)=[g(x)]^5$. Find $H'(2)$

d. $H(x)=g(x^2-7)$. Find $H'(3)$

e. $H(x)=f(g(x))$. Find $H'(5)$

Now let's consider the ***Numerical Derivative*** handout

Suppose that the function $f(x)$ and its first derivative have the following values at x = 0 and x = 1

x	$f(x)$	$f'(x)$
0	9	-2
1	-3	$\frac{1}{5}$

Find the first derivative of the following combinations at the given values of x

(a) $\sqrt{x}\ f(x)$ at x = 1

PRODUCT RULE

$$\frac{d}{dx}\left[\sqrt{x}\ f(x)\right]\Big|_{x=1}$$

$$= \frac{1}{2\sqrt{x}} f(x) + f'(x)\cdot\sqrt{x}\ \Big|_{x=1}$$

$$= \frac{1}{2\sqrt{1}} f(1) + f'(1)\cdot\sqrt{1}$$

$$= \frac{1}{2}(-3) + \frac{1}{5}$$

(c) $\dfrac{f(x)}{1+\cos x}$ at x = 0

$$\frac{d}{dx}\left[\frac{f(x)}{1+\cos x}\right]\Bigg|_{x=0}$$

$$= \frac{\overset{\text{Low}}{(1+\cos x)}\,\overset{d\,\text{high}}{f'(x)} - \overset{\text{high}}{f(x)}\,\overset{d\,\text{Low}}{(-\sin x)}}{\underset{[\text{Low}]^2}{[1+\cos x]^2}}\Bigg|_{x=0}$$

$$= \frac{(2)(-2) - (9)(0)}{4} = -1$$

(d) $f^{2}(x)$ at x = 0

[Remember: $f^{2}(x) = f(x) \bullet f(x)$]

6

$$\frac{d}{dx}\left[f(x)\,f(x)\right]\Big|_{x=0}$$

$$= f'(x)\,f(x) + f'(x)\,f(x)\Big|_{x=0}$$

$$= (-2)(9) + (-2)(9)$$

$$= -36$$

(e) $\dfrac{1}{f(x)}$ at x = 1

$$\frac{d}{dx}\left[\frac{1}{f(x)}\right]\Bigg|_{x=1}$$

$$= \frac{f(x)\cdot(0) - 1\cdot f'(x)}{[f(x)]^2}\Bigg|_{x=1}$$

$$= \frac{-\left(\frac{1}{5}\right)}{(-3)^2}$$

Homework: on the ***Finding Numerical Derivatives*** handout
Do (a), (b), (c), (d), (g) **on the bottom of the page**