Making Functions Continuous

Consider: $f(x)=\dfrac{\sin x}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

$\lim\limits_{x\to 0} f(x) = 1$

$$g(x)=\begin{cases} \dfrac{\sin x}{x}, & x\neq 0 \\ a, & x=0 \end{cases}$$

What should the value of a be such that $g(x)$ is continuous for all real numbers? Well, we only know one calculus "trick" so we should probably use it.

To be continuous at $x=0$, what must be true?

$\lim\limits_{x\to 0} g(x) = g(0)$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$ $\quad g(0) = 1$

$a = 1$

Let's try some !

$$f(x)=\begin{cases}\dfrac{x^2-9}{x-3}, & x\neq 3\\ b, & x=3\end{cases}$$

Need

$f(3)=\lim_{x\to 3} f(x)$

$$\lim_{x\to 3}\frac{x^2-9}{x-3}$$

$$=\lim_{x\to 3}\frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$$

$$=\lim_{x\to 3}(x+3)$$

$$=6$$

Hence, let $b=6$

$$g(x)=\begin{cases} x^2-5x+3, & x\le 2 \\ ax+1, & x>2 \end{cases}$$

LEFT

RIGHT

Need $\lim_{x\to 2} g(x)=g(2)$

$g(2)=-3$

$$\lim_{x\to 2^-} g(x) = -3$$

$$\lim_{x\to 2^+} g(x) = 2a+1$$

$$-3 = 2a+1$$

$$-4 = 2a$$

$$-2 = a$$

$$g(x)=\begin{cases} x^2-5x+3, & x\le 2 \\ -2x+1, & x>2 \end{cases}$$

$$h(x) = \begin{cases} \dfrac{k}{x^2}, & x < -2 \\ 9 - x^2, & x \geq -2 \end{cases}$$

Need

$$h(-2) = \lim_{x \to -2} h(x)$$

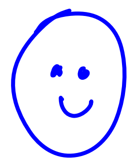
$$\lim_{x \to -2^-} h(x) = \frac{K}{4}$$

$$h(2) = 5$$

$$\lim_{x \to -2^+} h(x) = 5$$

$$5 = \frac{K}{4}$$

$$20 = K$$



Slightly harder:

$$f(x)=\begin{cases} x^2-5x+3, & x<-1 \\ ax+b, & -1\le x\le 4 \\ 11-3x, & x>4 \end{cases}$$

$-2x+7$

need

$f(-1)=\lim_{x\to -1} f(x)$

$f(4)=\lim_{x\to 4} f(x)$

$\lim_{x\to -1^-} f(x) = 9$

$\lim_{x\to -1^+} f(x) = -a+b$

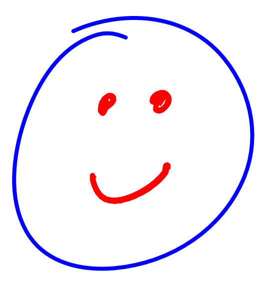
$9=-a+b$

$\lim_{x\to 4^-} f(x) = 4a+b$

$\lim_{x\to 4^+} f(x) = -1$

$-1 = 4a+b$

$$\begin{aligned} 9 &= -a+b \\ -1 &= 4a+b \\ \hline 10 &= -5a \\ -2 &= a \\ 7 &= b \end{aligned}$$



$$g(x)=\begin{cases} cx+1, & x\le 3 \\ cx^2-1, & x>3 \end{cases}$$

Need $g(3)=\lim_{x\to 3} g(x)$

$\lim_{x\to 3^-} g(x) = 3c+1$

$\lim_{x\to 3^+} g(x) = 9c-1$

$3c+1 = 9c-1$

$\frac{1}{3} = c$

Slightly different:

$$h(x)=\begin{cases} \dfrac{x^2-a^2}{x-a}, & x\ne a \\ 2, & x=a \end{cases}$$

$\lim_{x\to 3} g(x)$ should equal 2

need $h(a)=\lim_{x\to a} h(x)$

$h(a)=2$

$2=2a$

$1=a$

$$\lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} \frac{(x+a)\cancel{(x-a)}}{\cancel{x-a}} = \lim_{x\to a} (x+a) = 2a$$

Hence, $h(x)=\begin{cases} \dfrac{x^2-1}{x-1}, & x\ne 1 \\ 2, & x=1 \end{cases}$

Need more examples? Go to:
http://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/continuitydirectory/Continuity.html

♪ Remember to always find the one-sided limits for the x – value in question. You may also want to check your solution by graphing it.
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