How to find a limit if direct substitution fails [and you do not have a calculator]

If we use direct substitution and obtain the indeterminate form $\frac{0}{0}$, then it **may** be possible to do something algebraic. See page 62, Theorem 1.7.

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

If we let $x=2$, then we get an indeterminate form. Let's consider using Theorem 1.7.

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$ Factor, then cancel

$= \lim_{x \to 2} \frac{1}{x+2}$ Apply Theorem 1.7

$= \frac{1}{2+2}$ Now use direct substitution

$= \frac{1}{4}$ Simplify [check with TI]

Find: $\lim\limits_{x \to 2} \dfrac{x^2 - 3x + 2}{x - 2}$

$= \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x-1)}{\cancel{x-2}}$

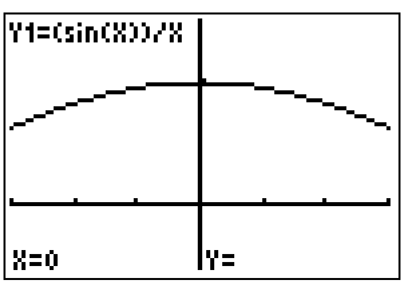
$= \lim\limits_{x \to 2} (x-1)$

$= 1$

Rationalizing also works – see page 64

Consider:

$\lim\limits_{x \to 0} \dfrac{\sin x}{x}$ If we use direct substitution, then we get the indeterminate form $\dfrac{0}{0}$, but nothing algebraic can be done. Let's use our TI to see if a limit exists.

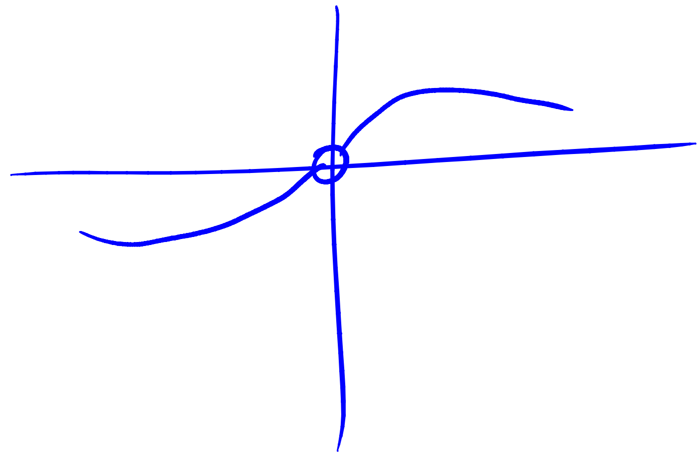


The graph looks continuous at x = 0 but it is not!
Use a table to find the limit.

$\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$

Find:

$$\lim_{x \to 0} \frac{1-\cos x}{x} = 0$$



♪ There is another analytic method that we can use to find these limits but we need to learn the next chapter in order to do it.

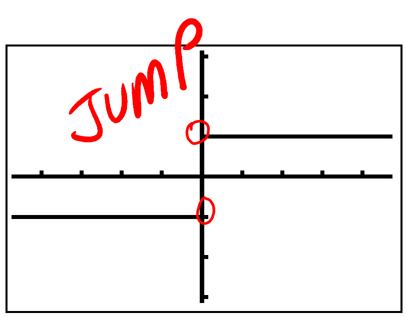
$$|x| = \begin{cases} -x, & x<0 \\ x, & x \geq 0 \end{cases}$$

Consider:

$$\lim_{x \to 0} \frac{|x|}{x}$$

Does direct substitution work?

How about a graph?



$$\lim_{x \to 0^-} f(x) = -1$$

$$\lim_{x \to 0^+} f(x) = 1$$

$$\lim_{x \to 0} \frac{|x|}{x} = \text{dne}$$

Which of the following can be done with direct substitution?

(1) $\lim_{x\to 0} \tan x$ yes

(2) $\lim_{x\to 0} \frac{\sin(2x)}{2x}$ NO TI

(3) $\lim_{x\to 0} \frac{x-1}{x^2-1}$ yes

(4) $\lim_{x\to 5} \frac{x^2-25}{x-5}$ NO

ALGEBRAIC

$$\lim_{x\to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}}$$

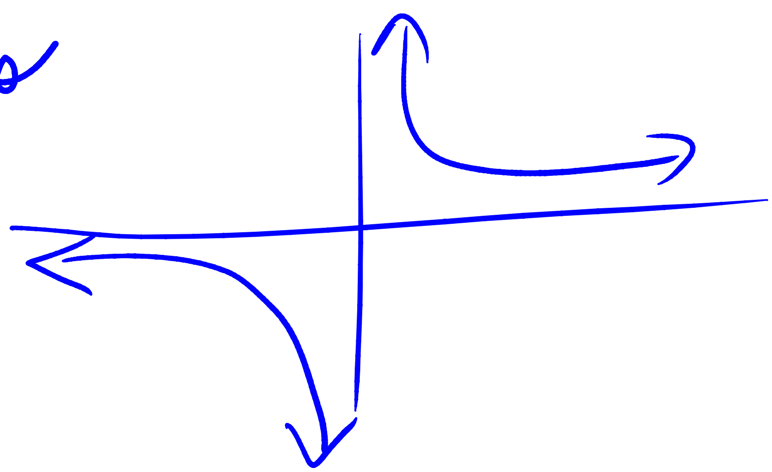
$$= \lim_{x\to 5} (x+5) = 10$$

(5) $\lim_{x\to 2} \frac{|x|}{x}$ yes

(6) $\lim_{x\to 1} \frac{x^2+2x+3}{x^2+5}$ yes

(7) $\lim_{x\to 0} \frac{x}{\cos x}$ yes

(8) $\lim_{x\to 0} \frac{1}{x}$ NO



$\Delta x \neq x$

Consider:

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} \qquad \text{[think of } \Delta x \text{ as } h\text{]}$$

NO

Does direct substitution work? If not, can we do something algebraic?

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

EXPAND

$$= \lim_{\Delta x \to 0} \frac{\left(\cancel{x^2} + 2x\Delta x + (\Delta x)^2\right) \cancel{- x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

FACTOR NUMERATOR

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

DIRECT SUBSTITUTION

♪ The Squeeze Theorem is not part of the AP curriculum

Homework on page 68: [re-read 1.3]

Do analytically: # 45, 46, 47, 48, 49, 59

Do graphically [show graph] # 67, 71