

# A PREVIEW OF CALCULUS

Calculus = Mathematics of Change

In real-life: velocity, acceleration, rate of change

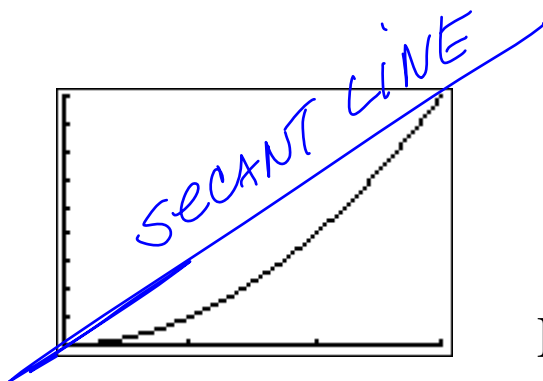
If you drive 40 miles in one hour, then your average rate of change = 40 mph. But what was your rate at time = 13 minutes?

Consider average rate of change:

Let  $f(x) = x^2$  on  $[0, 3]$

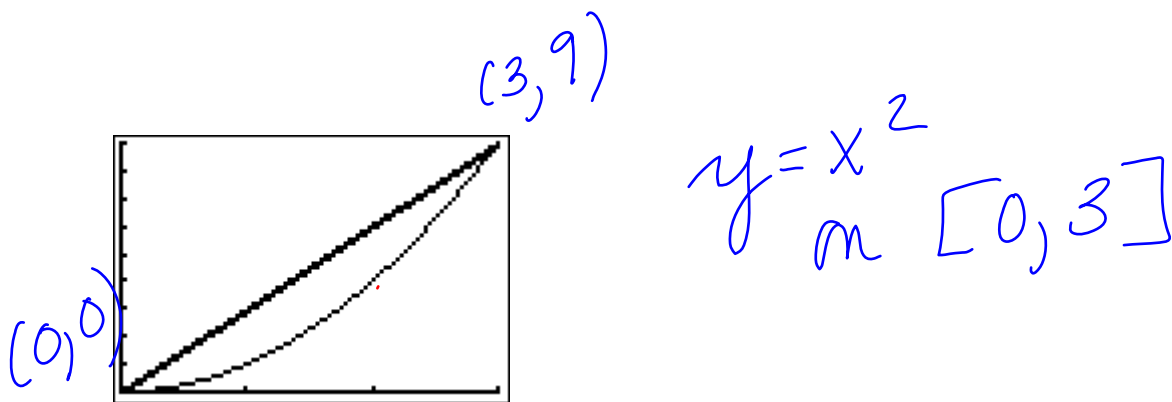
Average rate of change = slope of secant

Average rate of change =  $m_{\text{sec}}$



Let's draw in the secant

The *secant line* is a line which connects two points on the curve.



Now find the slope of the secant line

Average rate of change =  $m_{\text{sec}}$

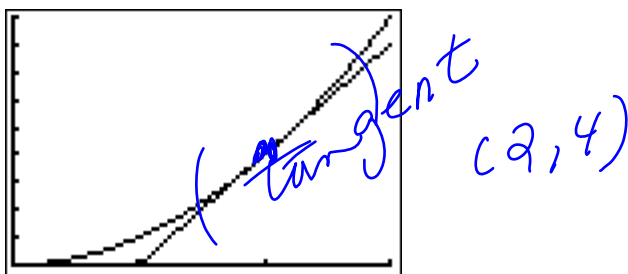
$$m_{\text{sec}} = \frac{f(3) - f(0)}{3 - 0}$$

$$m_{\text{sec}} = 3$$

What if we wanted the *instantaneous rate of change* at  $x = 2$ ?

Instantaneous rate of change = slope of a tangent to the curve

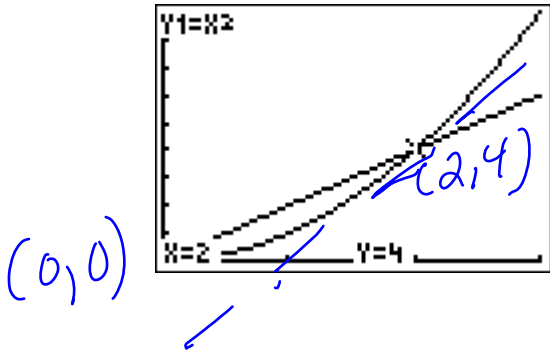
Instantaneous rate of change =  $m_{\text{tan}}$



This is what the tangent looks like at  $x = 2$ .

We could try to find  $m_{\text{sec}}$ .

Average rate of change =  $m_{\text{sec}}$  on  $[0, 2]$

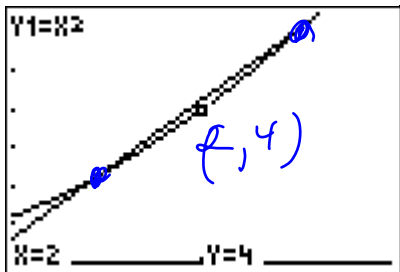


$$f(x) = x^2$$

$$m_{\text{sec}} = \frac{f(2) - f(0)}{2 - 0} \\ = 2$$

How about changing the interval to  $[1.5, 2.5]$ ?

Here is a close-up of the curve on this interval.

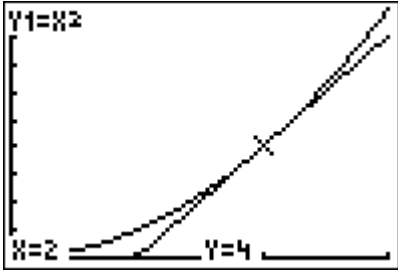


$$f(x) = x^2$$

Notice what is happening as we zoom in near  $x = 2$ .

$$\text{ARofC} = m_{\text{sec}} = \frac{f(2.5) - f(1.5)}{2.5 - 1.5} \\ = \frac{6.25 - 2.25}{1} \\ = 4$$

Hmm! How about changing the interval to [1.9, 2.1]?



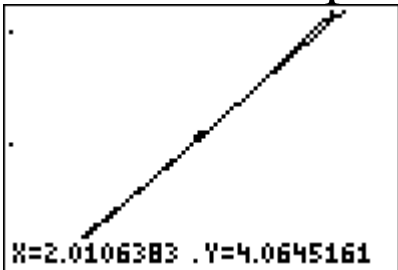
$$f(x) = x^2$$

Looks pretty good!

$$\begin{aligned} m_{\text{sec}} &= \frac{f(2.1) - f(1.9)}{2.1 - 1.9} \\ &= \frac{4.41 - 3.61}{.2} = 4 \end{aligned}$$

Here is a zoomed-in look near  $x = 2$ :

This shows a portion of the curve and the secant line.



What do you notice?

ALMOST the SAME

Our initial calculus goal will be to find instantaneous rates of change [  $m_{\text{tan}}$  ] and for that we will need limits!

## Limits

$\lim_{x \rightarrow c} f(x) = L$  where  $c, L \in$  real numbers

"element of"

*Informal definition of a limit:*

As  $x$  becomes arbitrarily close to some  $c$ ,  $c \in$  reals, then the function,  $f(x)$ , becomes arbitrarily close to  $L$ ,  $L \in$  reals. In other words, a predicted  $y$ -value.

♪ A limit is a  $y$ -value, a limit is a  $y$ -value, a limit is a  $y$ -value, ...

There are numerous ways to find a limit.

Our first limit problem:

Consider  $f(x) = \frac{x^2 - 3x + 2}{x - 2}$  and find  $\lim_{x \rightarrow 2} f(x)$

$$f(2) = \frac{2^2 - 3(2) + 2}{2 - 2} = \frac{0}{0}$$

What happens if you substitute  $x = 2$  into  $f(x)$ ?

ACK!  
undefined

How about making a table of values near  $x = 2$ ?

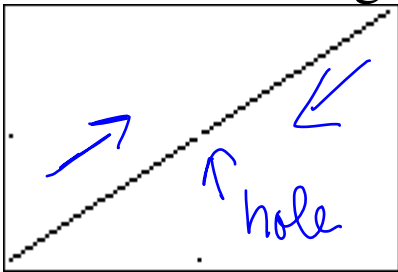
X	Y <sub>1</sub>	
1.8	.8	
1.9	.9	
1.95	.95	
2.05	1.05	
2.1	1.1	
2.2	1.2	

X=

What y-value does  $f(x)$  approach as  $x$  approaches 2?

|

What does the graph of  $f(x)$  look like near  $x = 2$ ?



graph of  $f(x)$  on  $[1.5, 2.5]$

$$\lim_{x \rightarrow 2} f(x) = 1$$

♪ Not all functions have limits at every value of  $x$   
[The domain of a function is very important!]

## Lucky Case Scenario:

For a continuous function [a function with NO “holes”, “jumps”, or vertical asymptotes]:

$$\lim_{x \rightarrow c} f(x) = f(c)$$

For example, find  $\lim_{x \rightarrow 3} x^2$

Since  $f(x) = x^2$  is a continuous function [well-behaved function], then  $\lim_{x \rightarrow 3} x^2 = f(3) = 9$

This method of finding a limit is called direct substitution.

$$\lim_{x \rightarrow 0} (x+1) = 0+1 = 1$$

$$\lim_{x \rightarrow 4} x^3 = 4^3 = 64$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \tan x = \tan\left(\frac{\pi}{4}\right) = 1$$

What if direct substitution does not work?

Don't PANIC!

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-4}$$

$$f(2) = \frac{0}{0} \text{ ACK!}$$

We could try using a table again

X	Y1
1.9	.25641
1.99	.25063
1.9999	.25001
2	ERROR
2.001	.24994
2.01	.24938

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-4} = \frac{1}{4}$$

Look at the graph of  $f(x) = \frac{x-2}{x^2-4}$ . What type of discontinuity occurs at  $x = 2$ ?

REMOVABLE DISCONTINUITY

Try:

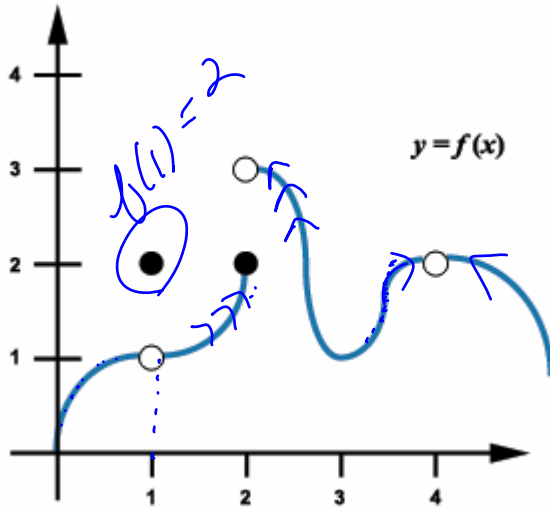
$$\lim_{x \rightarrow -3} \frac{\sqrt{1-x} - 2}{x+3}$$

$$\lim_{x \rightarrow -3} \frac{\sqrt{1-x} - 2}{x+3} = -\frac{1}{4}$$

[JUST ONE POINT HAS BEEN REMOVED]

$$f(-3) = \frac{\sqrt{1-(-3)} - 2}{-3+3} = \frac{0}{0} \text{ ACK! EEEK!}$$

Another way to find a limit is graphically – in other words, just look at it!



[from: <http://curvebank.calstatela.edu>]

Find the following:

$$\lim_{x \rightarrow 1} f(x) = 1$$

$$\text{What is } f(1)? = 2$$

$$\lim_{x \rightarrow 4} f(x)$$

$$\text{What is } f(4)? = 2$$

$$\lim_{x \rightarrow 2} f(x) \text{ DOES NOT EXIST}$$

because from the left we expect  $y=2$  BUT from the right we expect  $y=3$

What is  $f(2)$ ?

$$f(2) = 2$$

## Limits that fail to exist

JUMP DISCONTINUITY

1.  $f(x)$  approaches a different number from the right side of  $c$  than it approaches from the left side

2.  $f(x)$  increases or decreases without bound as  $x$  approaches  $c$

VERTICAL ASYMPTOTE  $\lim_{x \rightarrow \frac{\pi}{2}} \tan x$

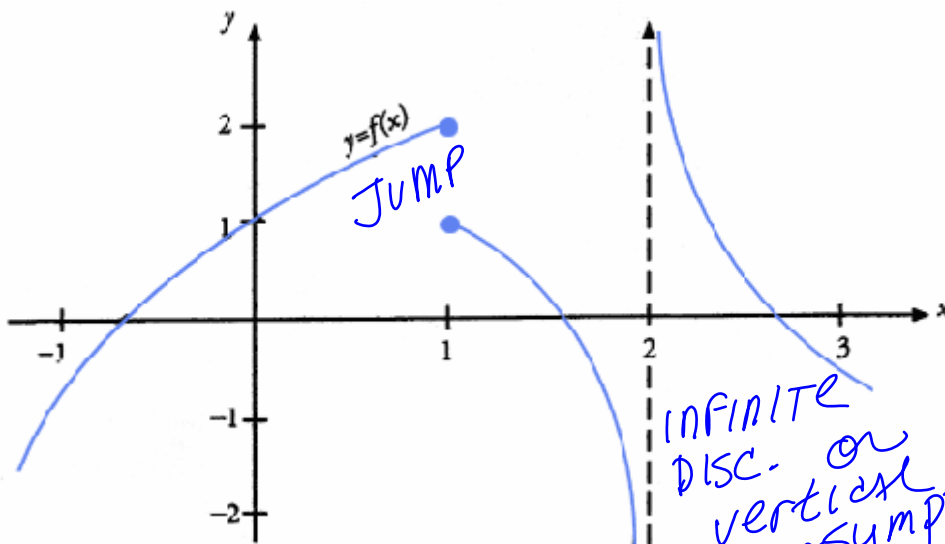
3.  $f(x)$  oscillates between two fixed values as  $x$  approaches  $c$

$\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$

See page 55 #14, 16, and 18

v.a.      (      v.a.

OK



<http://curvebank>

Where do the limits not exist for this function and why?

$$\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right) \quad \text{see page 51}$$

OSCILLATING

♪ Calc AB does not cover  $\epsilon - \delta$

**Homework:** read 1.1 and 1.2; do pages 54, 55 # 3, 5, 11, 13, 15, 17, 19, 23 *This assignment concentrates on finding limits using a table of values, or a graph* [Please write the problem using standard mathematical notation, if a table is given, then write down the table, if a graph is given, then sketch the graph]

On the web: Another great look at limits can be found at:

[http://www.brookscole.com/math\\_d/special\\_features/calclabs/sv4e\\_ti\\_83/ch03.pdf](http://www.brookscole.com/math_d/special_features/calclabs/sv4e_ti_83/ch03.pdf)