Some More Weird Stuff for Chapter Two

From our handout:

12. From: http://www.wildstrom.com

Use the following table of values to compute the requested derivatives.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	−3	4	5	−1
2	5	1	3	7
3	−2	2	1	8
4	6	5	2	−3
5	9	3	4	−4

a. $H(x)=5f(x)-3g(x)$. Find $H'(1)$

b. $H(x)=\dfrac{f(x)}{g(x)}$. Find $H'(4)$

c. $H(x)=[g(x)]^5$. Find $H'(2)$

d. $H(x)=g(x^2-7)$. Find $H'(3)$

e. $H(x)=f(g(x))$. Find $H'(5)$

c) $H(x) = [g(x)]^5$

$H(x) = u^5$

$u = g(x)$

$\frac{du}{dx} = g'(x)$

$H'(x) = 5u^4 \frac{du}{dx}$

$H'(x) = 5[g(x)]^4 g'(x)$

d) $H(x) = g(x^2-7)$

$H(x) = g(u)$

$H'(x) = g'(u)\frac{du}{dx}$

$H'(x) = [g'(x^2-7)][2x]$

$u = x^2-7$

$\frac{du}{dx} = 2x$

e) $H(x) = f(g(x))$

$H(x) = f(u)$

$u = g(x)$

$$H'(x) = f'(u)\frac{du}{dx} \qquad \frac{du}{dx} = g'(x)$$

$$H'(x) = f(g(x))g'(x)$$

15. Ms. McCleary has the unfortunate delight of teaching an AP Calculus class during the 1st period of the school day. For many reasons most of her students have a difficult time staying awake in class. Her evaluator took the following data one day while observing the class.

Elapsed class time [minutes]	5	10	15	20	25	30	35	40	45
Number of students who were awake	29	26	22	17	14	11	19	24	28

(a) The number of students awake is a function of time, t. Call this function $A(t)$. Using the data in the table, approximate $A'(15)$. What does this represent?

$$A'(15) \approx \frac{A(15) - A(10)}{15 - 10} = -\frac{4}{5}$$

every 5 minutes 4 students fall asleep

$$A'(15) \approx \frac{A(20) - A(15)}{20 - 15} = \frac{-5}{5}$$

$$A'(15) = \frac{A(20) - A(10)}{20 - 10} = \frac{-9}{10}$$

(b) Use your answer from part (a) to write the equation of the tangent line at $t = 15$.

$$y - 22 = -\frac{4}{5}(x - 15)$$

(c) Use your answer from part (b) to estimate $A(17)$

let $x = 17$

$$y - 22 = -.8(17 - 15)$$

$$y = 20.4$$

2005 AB5B [we'll skip part (d)]

Consider the curve given by: $y^2 = 2 + xy$

(A) Show that $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$

$$\frac{d}{dx} y^2 = \frac{d}{dx} 2 + \frac{d}{dx} xy$$

$$2y \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - x) = y$$

$$\frac{dy}{dx} = \frac{y}{2y - x}$$

(B) Find all points (x, y) on the curve when the line tangent to the curve has slope $\frac{1}{2}$.

$$\frac{dy}{dx} = \frac{1}{2}$$

$$\frac{y}{2y - x} = \frac{1}{2}$$

$$2y = 2y - x$$

$$0 = x$$

$y^2 = 2 + xy$ let $x = 0$

$y^2 = 2 + (0)y$

$y = \pm\sqrt{2}$

POINTS: $(0, \sqrt{2})$, $(0, -\sqrt{2})$

(C) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

$$\frac{dy}{dx} = 0$$

$$\frac{y}{2y - x} = 0 \quad \text{IF } y = 0$$

let $y = 0$ $\quad y^2 = 2 + xy$

$0^2 = 2 + (0)x$

$0 = 2$ This is NEVER TRUE $0 \neq 2$

Hence, the curve HAS NO HORIZONTAL TANGENTS