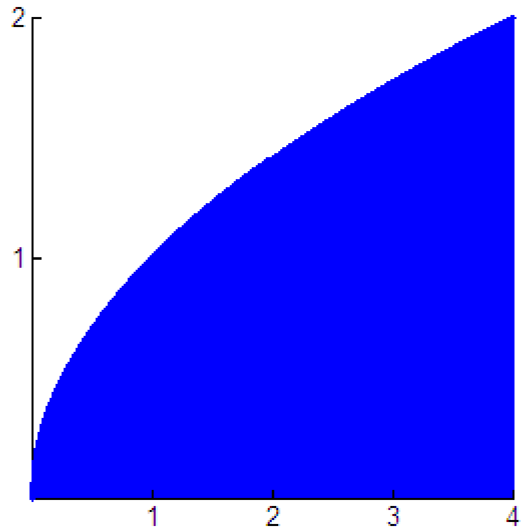
Volume of a Solid of Revolution

A solid of revolution is a solid that is generated by revolving a plane region about a line that lies in the same plane as the region; the line is called the axis of revolution.

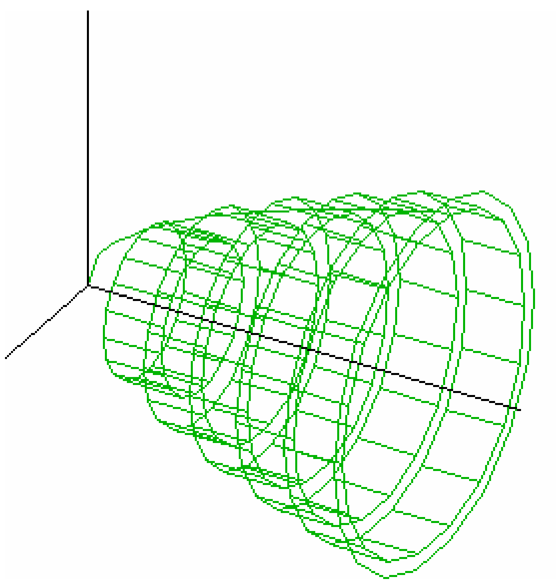
When revolving a region we take the region around the axis 360 degrees. Hence the cross section of a "slice" is always a circle. If the region is adjacent to the axis of revolution, then the resulting figure is solid – i.e. no "holes". If any part of the region is not adjacent to the axis of revolution, then the resulting figure will have a "hole". [Think of a donut!]

In most Calculus books the methods that we will be using are called "the Disk Method" and "the Washer Method" because our cross sections will look like a disk [a circle] or a washer.

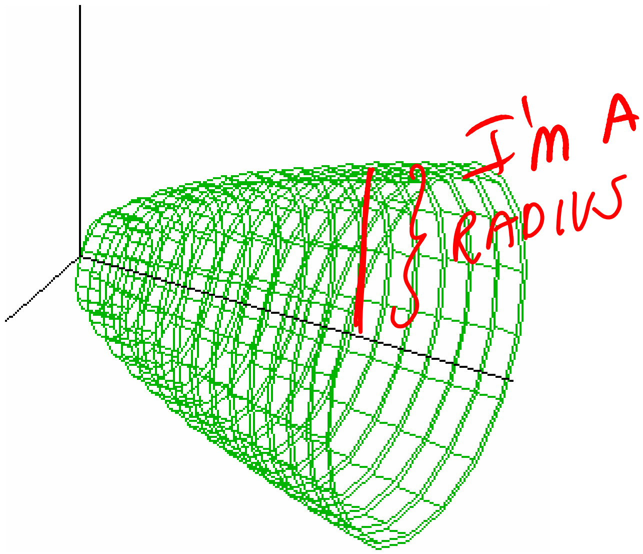
Let's see what this looks like:
Let our base region be the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$



Go to and animate: http://www.ies.co.jp/math/products/calc/applets/rotate/rotate.html



Here is the solid with a few disks shown.



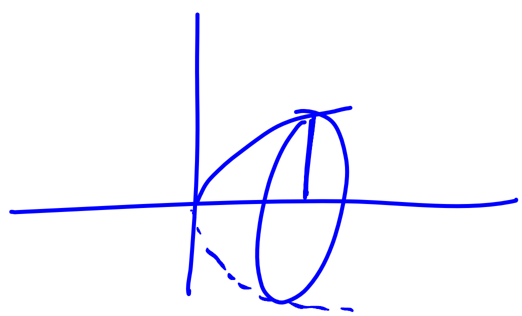
Here's with a lot of disks shown.

Volume = Area of a cross section * height
Where the area of a cross section = πr^2 and $r = \sqrt{x}$

$$V = \int_0^4 \pi r^2 \, dx$$

$$V = \pi \int_0^4 \left(\sqrt{x}\right)^2 dx$$

$$V = 8\pi (units)^3$$

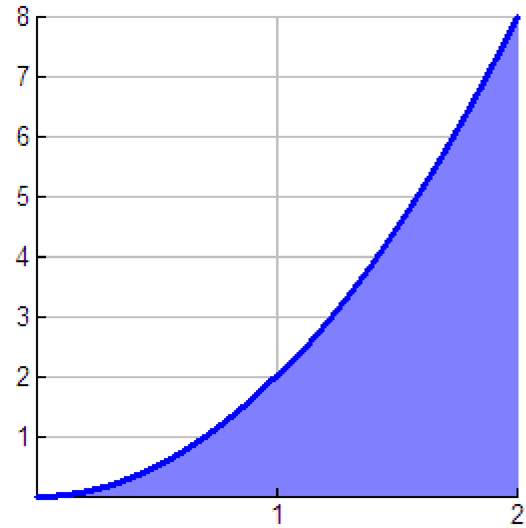


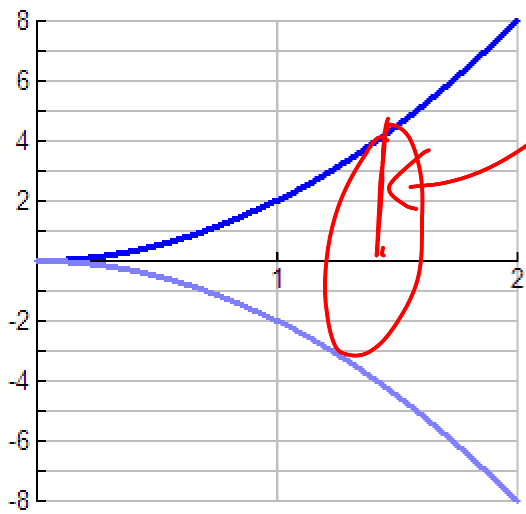
When revolving a region about the x-axis or y-axis, our cross sections will always be circles.

Let's try some:

Revolve the region defined by $y = 2x^2$, $y = 0$, $x = 2$ about the x-axis

Here's the region:





I'M A RADIUS

$A(x) = \pi r^2$

$$V = \int_0^2 \pi r^2 \, dx \text{ where } r = 2x^2$$

$$V = \pi \int_0^2 (2x^2)^2 \, dx$$

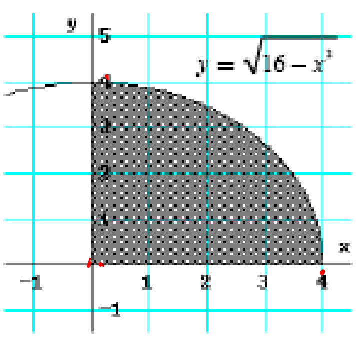
$\approx 25.6\pi$ CUBIC UNITS

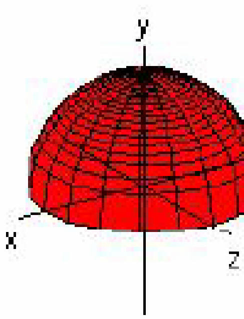
How about a little song?
[Lyrics by D.F. McCleary, Melody is "The Wheels on the Bus"]

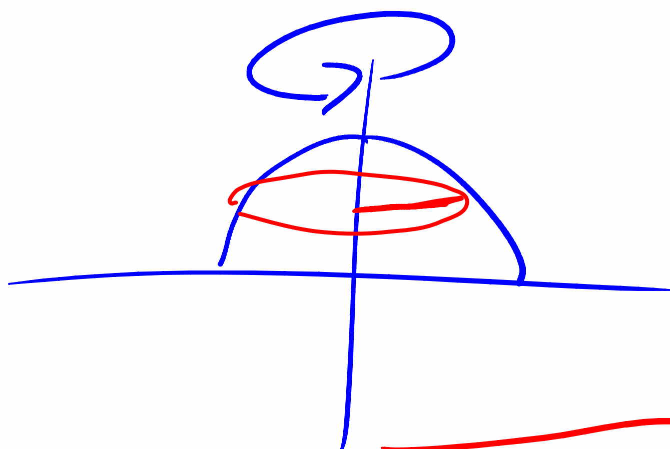
The region on the graph goes round and round
Round and Round
Round and Round
The region on the graph goes round and round
Each slice is a CIRCLE!

Revolving a region about the y- axis
From: Mr. Leckie's website – http://chaoticgolf.com

Example: Find the volume of the solid formed by revolving the region about the y – axis. (Draw a representati
rectangular strip)







$$y = \sqrt{16 - x^2}$$
$$y^2 = 16 - x^2$$
$$x^2 = 16 - y^2$$
$$x = \sqrt{16 - y^2}$$

Volume [with vertical axis of revolution]

$A(y) = \pi r^2$

$$V = \pi \int_c^d [R(y)]^2 \, dy$$

c, d are y- bounds, and $R(y)$ is the radius in terms of y

In this case: $V = \pi \int_0^4 \left[\underbrace{\sqrt{16 - y^2}}_{r}\right]^2 dy$

$r = \sqrt{16 - y^2}$

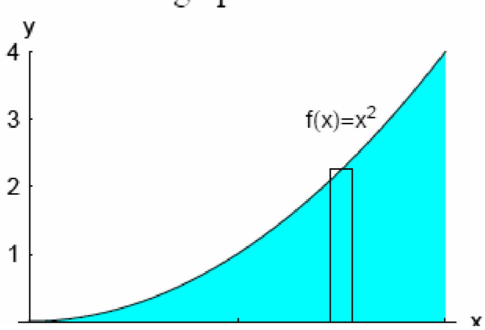
$$V = \pi \int_0^4 (16 - y^2)\, dy$$

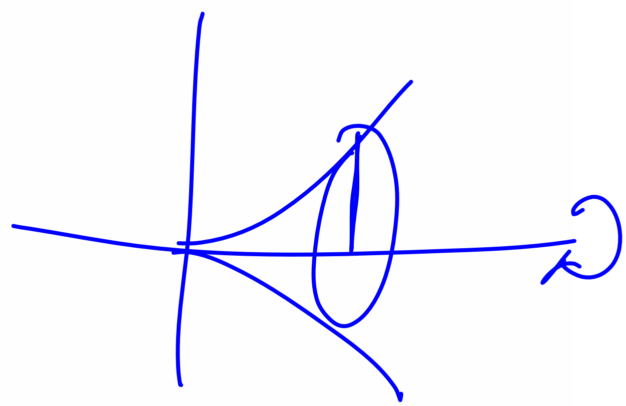
$$= \frac{128}{3}\pi \text{ c.u.}$$

Let's consider one more:
From: Mr. Zab's website

Example 6: Find the volume of the solid obtained when the region enclosed by the graph of f(x) = x^2, the x-axis, and the vertical lines x = 0 and x = 2 is revolved about the x-axis.

Solution: Look at the graph below.

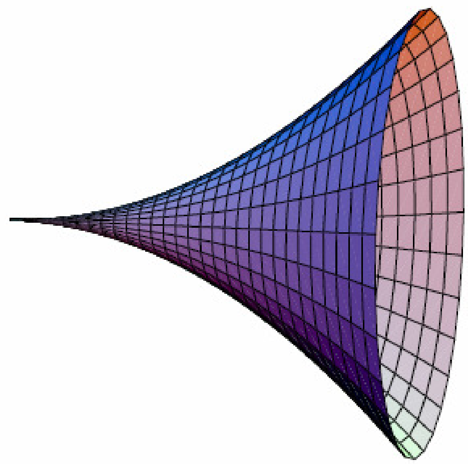




$r = x^2$

$A(x) = \pi r^2$

When this region is revolved about the x-axis it will sweep out a region in space that looks like a volcano on its side (or maybe the end of a trumpet).



$$V = \pi\int_0^2 (x^2)^2\,dx$$

$$= \pi\left(\frac{x^5}{5}\right)\Big|_0^2$$

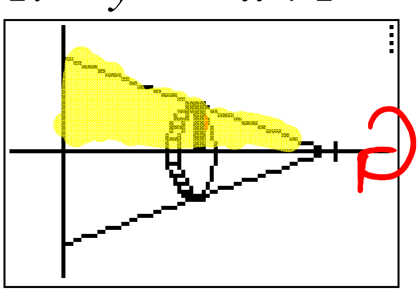
$$= \frac{32}{5}\pi \text{ c.u.}$$

$V = \pi\int_0^2 r^2 dx$ where $r = x^2$

Note: All of the solids formed today have no "holes". In our next class we will explore how to find the volume when the solids has a "hole" in it [sort of like a doughnut].

If time see page 463 Let's see what some of these look like.

1. $y = -x + 1$

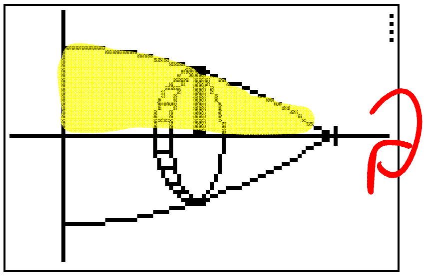


$$A(x) = \pi r^2$$

$$r = -x + 1$$

$$V = \pi\int_0^1 (-x+1)^2\,dx$$

2. $y = 4 - x^2$



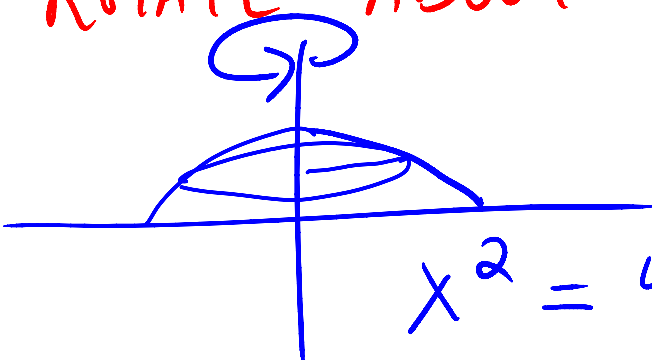
$A(x) = \pi r^2$

$r = 4 - x^2$

$V = \pi \int_0 (4-x^2)^2\,dx$

TAKE $y = 4 - x^2$ FROM $x=0$, $x=2$

ROTATE ABOUT y-AXIS



$A(y) = \pi r^2$

$r = \sqrt{4-y}$

$x^2 = 4 - y$

$x = \sqrt{4-y}$

$y = 0$

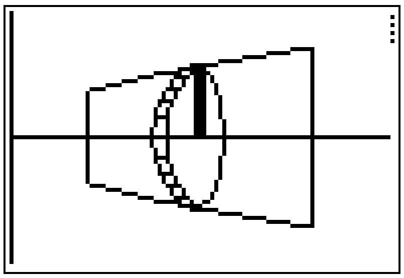
$y = 4$

$V = \pi \int_0^4 (\sqrt{4-y})^2\,dy$

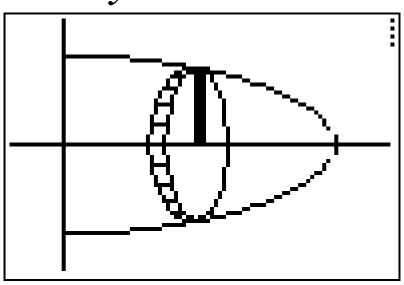
$= 8\pi$ c.u.

Homework: Read 7.2, do pages 463, 464 #3, 4, 7, 8, 9, 11A [set-up the integral and evaluate with your TI] Drawing a graph really helps. Here is what my graphs look like.

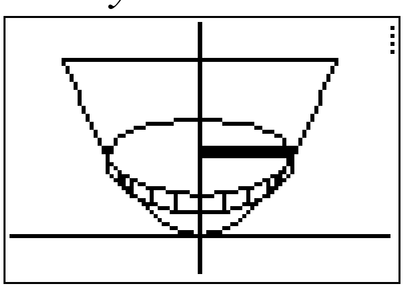
3. $y = \sqrt{x}$



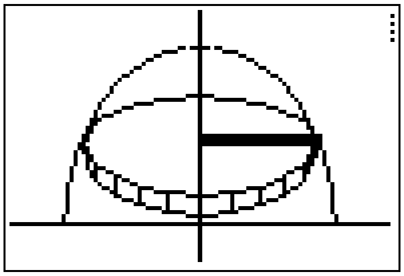
4. $y = \sqrt{9 - x^2}$



7. $y = x^2$ Note: You will need to rewrite!



8. $y = \sqrt{16 - x^2}$ Note: You will need to rewrite!



11a $\qquad y = \sqrt{x}$