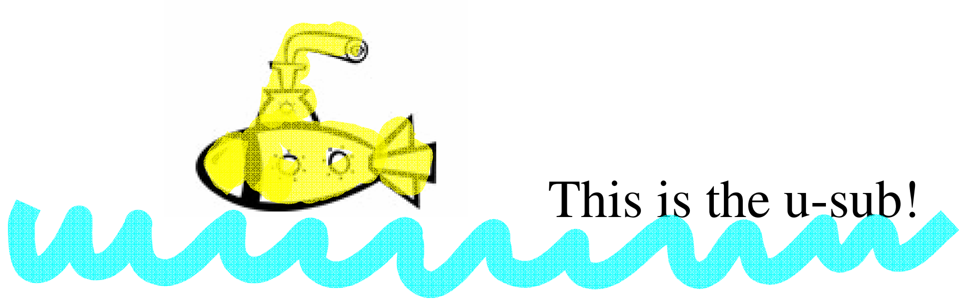
Using u-sub with Definite Integrals



There are two schools of thought on how to do this. I am an advocate of re-writing everything, including the upper and lower bounds, in terms of u and evaluating in terms of u.

Example

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 2\sin(2x)\,dx$$

Let $u = 2x$

Then $du = 2\,dx$

But our upper and lower bounds are $x-$values. We can re-write the bounds and then no longer worry about any $x-$values.

$u\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$ Our new lower bound

$u\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$ Our new upper bound

We can re-write our integral in terms of u

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 2\sin(2x)\,dx$$ will be rewritten as:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin u \;\; du = -\cos u \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= -\cos\frac{\pi}{2} - \left(-\cos\frac{\pi}{3}\right)$$

$$= \frac{1}{2}$$

Another Example:

$$\int_0^1 3x^2\left(x^3+1\right)^2 dx$$

$u = x^3+1$ ✓

$du = 3x^2\,dx$ ✓

$u(0) = 1$ ✓

$u(1) = 2$ ✓

$$= \int_1^2 u^2\,du$$

$$= \frac{u^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

Try:

$$\int_{-1}^{0} x\sqrt{1-x^2}\,dx$$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$u(-1) = 0$

$u(0) = 1$

$$= -\frac{1}{2}\int_0^1 u^{\frac{1}{2}}\,du = \boxed{-\frac{1}{\cancel{2}}\boxed{\frac{\cancel{2}}{3}}}\left[u^{\frac{3}{2}}\right]\Big|_0^1$$

$$= -\frac{1}{3}[1-0]$$

$$= -\frac{1}{3}$$

$$\int_0^{\frac{\pi}{6}} \cos(2x)\,dx$$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$u(0) = 0$

$u\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$

$$= \frac{1}{2}\int_0^{\frac{\pi}{3}} \cos u\,du$$

$$= \frac{1}{2}\left[\sin u\Big|_0^{\frac{\pi}{3}}\right]$$

$$= \frac{1}{2}\left[\frac{\sqrt{3}}{2} - 0\right] = \frac{\sqrt{3}}{4}$$

$$\int_0^3 \frac{1}{\sqrt{x+1}}\,dx$$

$u = x+1$

$du = dx$

$u(0) = 1$

$u(3) = 4$

$$= \int_1^4 u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}}\Big|_1^4$$

$$= 2\sqrt{4} - 2\sqrt{1}$$

$$= 2$$

$$\int_0^{\frac{\pi}{4}} \sin^2 x \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$u(0) = 0$

$u(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$$= \int_0^{\frac{1}{\sqrt{2}}} u^2 \, du$$

$$= \frac{u^3}{3} \Big|_0^{\frac{1}{\sqrt{2}}} = \frac{\left(\frac{1}{\sqrt{2}}\right)^3}{3} - 0$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{1}{3} = \frac{1}{6\sqrt{2}}$$

$$\int_1^4 \frac{\cos(\sqrt{x})}{\sqrt{x}} \, dx$$

$u = \sqrt{x} = x^{\frac{1}{2}}$ ✓

$du = \frac{1}{2} x^{-\frac{1}{2}} dx = \frac{1}{2\sqrt{x}} dx$

$2\,du = \frac{1}{\sqrt{x}} dx$ ✓

$u(1) = 1$ ✓

$u(4) = 2$ ✓

$$= 2\int_1^2 \cos u \, du$$

$$= 2\sin u \Big|_1^2$$

$$= 2\sin 2 - 2\sin 1$$

$$\int_1^3 3\,g'(3x)\,dx$$

$u = 3x$

$du = 3\,dx$

$u(1) = 3$

$u(3) = 9$

$$= \int_3^9 g'(u)\,du$$

$$= g(u)\Big|_3^9$$

$$= g(9) - g(3)$$

Homework: page 305 # 71, 73, 75, 76, 81 [you must show all steps with proper notation]