Diff E-Q
Calculus Lyrics by Denise Fuji McCleary
[Sung to "*Can't Help Falling in Love*" by G.D. Weiss which was made famous by Elvis (Presley not Costello)]

Diff EQs
Are such fun to solve
We could start drawing a nice slope field
Diff EQs
Need just four steps
If we could just remember those four steps

First we separate
Then we integrate
Now we solve for C
Then we just – we solve for y

Diff EQs
Are such fun to solve
Just remember to do those four small steps
Just remember to do those four small steps

From our friends at ETS College Board

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$

Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$ AND state its domain.

$$\frac{dy}{dx} = \frac{x}{y}$$

$$y\,dy = x\,dx$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + C$$

$(3, -1)$

$$(-1)^2 = (3)^2 + C \qquad \therefore C = -8$$

$$y^2 = x^2 - 8$$

$$y = -\sqrt{x^2 - 8}$$

$$x > \sqrt{8}$$

2004 AB6 (in your packet)

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$

$$\frac{dy}{dx} = x^2(y-1)$$

$$\frac{1}{y-1}\,dy = x^2\,dx$$

$$\int \frac{1}{y-1}\,dy = \int x^2\,dx$$

$$\ln|y-1| = \frac{x^3}{3} + C$$

$$e^{\ln|y-1|} = e^{\frac{x^3}{3}+C}$$

use (0, 3)

$$y-1 = Ce^{\frac{x^3}{3}}$$

$$3-1 = Ce^{0} \quad \therefore C = 2$$

$$y-1 = 2e^{\frac{x^3}{3}}$$

$$y = 1 + 2e^{\frac{x^3}{3}}$$