Basic Differentiation Rules

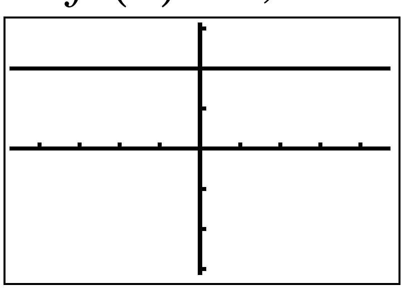
Consider $f(x)=\sqrt{x}$
Well, if we tried to find $f'(x)$ with our limit definition, it might get a little scary.
Using what we know from yesterday, it would look like:

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}$$

I'm scared already. There must be a better way!
Let's look at what we have already discovered using the limit definition of derivative:

Yesterday we found that the derivative of a constant equals zero.
Why?
If $f(x)=c$, $c\in$ Reals, then $f'(x)=0$



$\Delta y=?$

SINCE A derivative IS A "RATE OF Δ" AND y DOES NOT CHANGE

$f(x) = 11$	$f'(x) =$ 0
$f(x) = \pi$	$f'(x) =$ 0
$f(x) = e$	$f'(x) =$ 0
$f(x) = -711$	$f'(x) =$ 0
$f(x) = (\pi e)^2$	$f'(x) =$ 0

What we know about derivatives so far:

The derivative of a linear function is its slope.

$f(x) = 12x - 3$

$f'(x) = 12$

$f(x) = x$ linear $\qquad f'(x) = 1$

$f(x) = x^2 \qquad f'(x) = 2x$

$f(x) = x^3 \qquad f'(x) = 3x^2$

exponents of $f'(x)$ decrease by 1

$f(x) = \sqrt{x} \; or \; x^{\frac{1}{2}} \qquad f'(x) = \frac{1}{2\sqrt{x}} \; or \; \frac{1}{2} x^{\frac{-1}{2}}$

Hmmm! I sense a pattern!

If $f(x) = x^n$, $n \neq 0$, then $f'(x) = nx^{n-1}$

This is called the **POWER RULE**.

Great! Now we don't have to do the limit definition of derivative. [But, we can not forget it either.]

Let's find some derivatives:

$f(x)$	$f'(x)$
x^4	$4x^3$
x^5	$5x^4$
x^{100}	$100x^{99}$
x^{-2}	$-2x^{-3}$
x^{-7}	$-7x^{-8}$
x^{-100}	$-100x^{-101}$

Now finding the equation of tangent lines has become a lot easier.

Please find the equation of the tangent line to $f(x) = \sqrt{x}$ at the point (1, 1).

We need $f'(1)$, the slope of the tangent line at $x = 1$

If $f(x) = x^{\frac{1}{2}}$, then $f'(x) = \frac{1}{2\sqrt{x}}$ or $\frac{1}{2}x^{\frac{-1}{2}}$.

So, $f'(1) = \frac{1}{2}$.

Hence, the equation of the tangent line is:

$$y - 1 = \frac{1}{2}(x - 1)$$

Some more differentiation rules

Constant Multiple Rule [think of the CMR for limits]

$$\frac{d}{dx}[c\,f(x)] = c\,f\,'(x) \quad \text{where } c \in \text{ Reals}$$

CMR in action

$$f(x) = 2x^3$$

$$f\,'(x) = 2 \bullet \frac{d}{dx}\left(x^3\right)$$

Hence, $f\,'(x) = 2 \bullet 3x^2$ or $6x^2$

$$g(x) = 7x^{11}$$

$$g\,'(x) = 7 \bullet \frac{d}{dx}\left(x^{11}\right)$$

$$g\,'(x) = 7 \bullet 11x^{10} \text{ or } 77x^{10}$$

Hmmm! Another pattern?

$$f(x) = cx^n \qquad c \in \mathbb{R}$$

$$f\,'(x) = cn\,x^{n-1} \qquad n \neq 0$$

For $n \neq 0$, $\frac{d}{dx}\left(cx^n\right) = cnx^{n-1}$ $\qquad c \in$ Reals

$f(x)$	$f'(x)$
$9x^{10}$	$90x^9$
$\frac{-2}{x} = -2x^{-1}$	$2x^{-2}$
$\frac{2x^5}{5} = \frac{2}{5}x^5$	$2x^4$

♪ Whenever you see a grouping symbol, such as the division bar, or radical, it is very helpful to re-write the function using rational exponents.

Sum and Difference Rules [think of the limit rules]

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}[x^3 + x^2] = \frac{d}{dx}(x^3) + \frac{d}{dx}(x^2)$$

$$= 3x^2 + 2x$$

$$\frac{d}{dx}[\sqrt{x} - x^4]$$

$$= \frac{d}{dx}\left(x^{\frac{1}{2}}\right) - \frac{d}{dx}(x^4)$$

$$= \frac{1}{2}x^{-\frac{1}{2}} - 4x^3$$

$$=\frac{1}{2\sqrt{x}}-4x^3$$

Now let's use all of the new, handy rules:

$f(x)$	$f'(x)$
$3x^2+7x^1$	$6x+7$
x^3+3x^2+3x+1	$3x^2+6x+3$
$17-x^{-2}$	$2x^{-3}$
$\pi x^2+\sqrt[3]{x}$ $=\pi x^2+x^{\frac{1}{3}}$	$2\pi x+\frac{1}{3}x^{-\frac{2}{3}}$

$\frac{d}{dx}17+\frac{d}{dx}-x^{-2}$
$=0+2x^{-3}$

Sometimes it pays to rewrite your function:

If $y=\frac{7}{11x}$, then $y'=?$

$y=\frac{7}{11}x^{-1}$

$y'=-\frac{7}{11}x^{-2}$

If $y=\frac{2}{(3x)^2}$, then $y'=?$

$y=\frac{2}{9x^2}$

$y=\frac{2}{9}x^{-2}$

$y'=-\frac{4}{9}x^{-3}$

If $y = \frac{\sqrt{x}}{x^2}$, then $y' = ?$

$x > 0$

SIMPLIFY

$y = x^{\frac{1}{2}} \cdot x^{-2}$

$y = x^{-\frac{3}{2}}$

$y' = -\frac{3}{2} x^{-\frac{5}{2}}$

If $y = (x+3)^2$, then $y' = ?$

$y = x^2 + 6x + 9$

$y' = 2x + 6$

If $y = \frac{3}{x^{-4}}$, then $y' = ?$

$y = \frac{3}{\frac{1}{x^4}}$

$y = 3x^4$

$y' = 12x^3$

If $y = ex^2$, then $y' = ?$

$y' = 2ex$

Some Trigonometric Derivatives

Please graph the following on $[-2\pi, 2\pi]$:

$y_1 = \sin x$

$y_2 = nDeriv(y_1, x, x)$

Guess [based on the graph]

$\frac{d}{dx}(\sin x) = ?$ cos x

Now let $y_1 = \cos x$ and use the same y_2

Guess [based on the graph]

$\frac{d}{dx}(\cos x) = ?$ - SIN X

These derivatives can be found using the limit definition of derivative and can be found on page 112.

The same basic differentiation rules apply. [Yay! We don't have to learn new ones!]

Now let's find some more derivatives:

$f(x)$	$f'(x)$
$7\sin x$	7cos x
$11\cos x$	−11 sin x
$5\sin x - 6\cos x$	5cos x + 6sin x

f(x) = ... –
f'(x) = ...

Homework: read 2.2 and do page 115 #3-17 odds
and page 116 #57-62 all ♪ f'(x)=0 for horizontal tangents.

Please use our new rules [please be mindful of your notation]

No need for limits yay! ☺