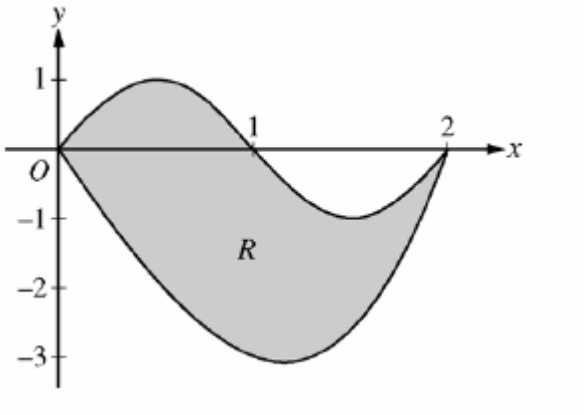
Lots and lots of Free Response Questions to Practice! [Chapters 6 and 7]
Please do your work on a separate piece of paper and observe the AP rules.

All of the following questions were found at:
http://www.collegeboard.com/student/testing/ap/prep_free.html#cala
These are the sole property of ETS College Board

2008AB1 [calculator]



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$ as shown in the figure above.

(a) Find the area of R

(b) The horizontal line $y = -2$ splits the region into two parts. Write, but do NOT evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the $x-$axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the $y-$axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

2004 AB3 [calculator]

A particle moves along the $y-$axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$.

(a) Find the acceleration of the particle at time $t = 2$

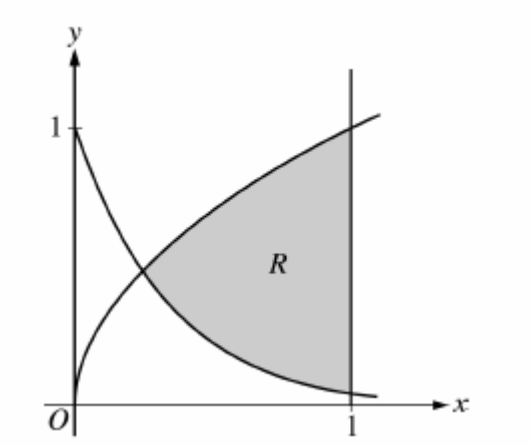
(b) Is the speed of the particle increasing or decreasing at time $t = 2$?

(c) Find the time $t \geq 0$ at which the particle reaches its highest point.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at $t = 2$?

[Remember to justify all solutions with CALCULUS!]

2003 AB1[calculator]



Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$ as shown in the figure above.

(a) Find the area of R

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the $x-$axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

2003 AB2 [calculator]

A particle moves along the $x-$axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$$

At time $t = 0$, the particle is at position $x = 1$.

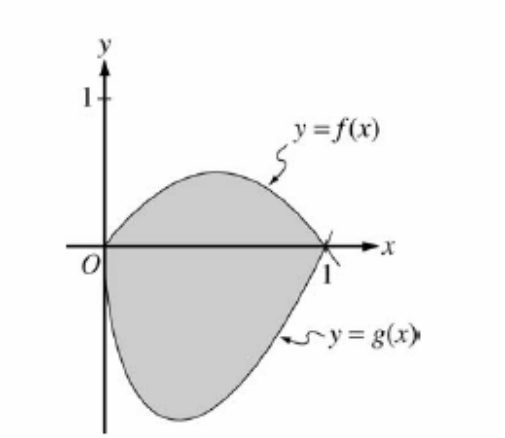
(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show all work that leads to your answer.

2004 AB2 [calculator]



Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$
The graphs of f and g are shown above.

(a) Find the area of the shaded region enclosed by the graphs of f and g

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$

(c) Let h be the function given by $h(x) = k\,x(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the $x-$axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k

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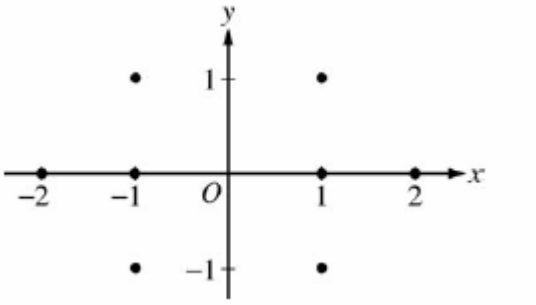
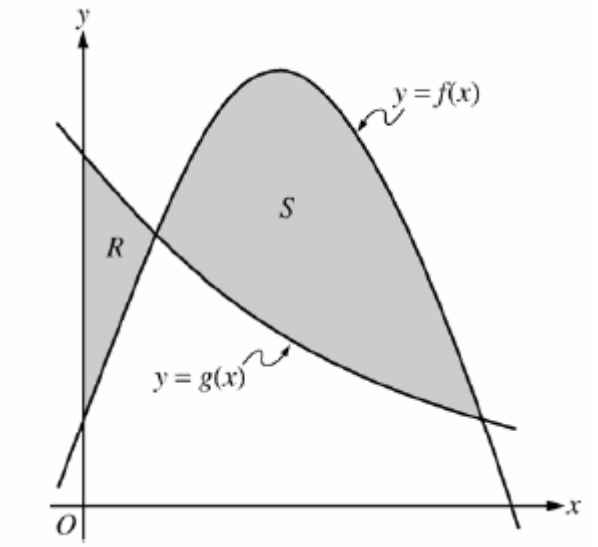
2006 AB 5 [non-calculator]

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$ where $x \neq 0$
On the axes provided below, sketch a slope field for the given differential equation at the eight points indicated.



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

2005 AB1 [calculator]



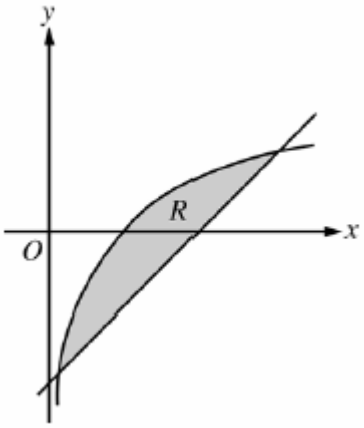
Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the region in the first quadrant enclosed by the $y-$axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g as shown above.

(a) Find the area of R

(b) Find the area of S

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$

2006 AB 1 [calculator]



Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $= x - 2$ as shown above.

(a) Find the area of R

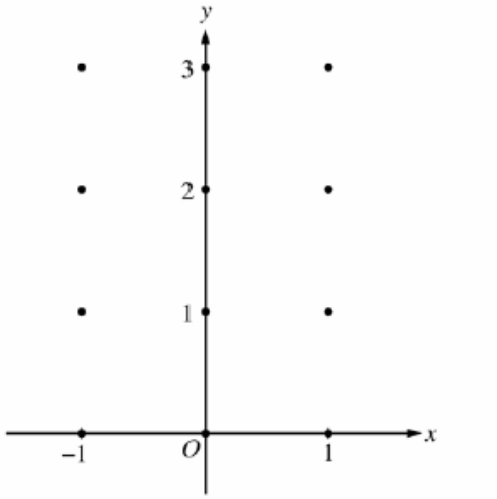
(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the $y-$axis.

2004 AB6 [non-calculator]

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$

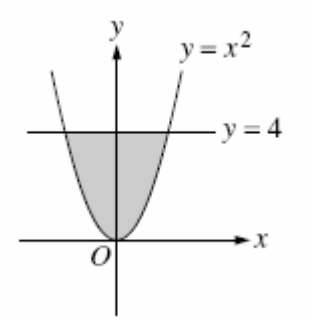
(a) On the axes provided below, sketch a slope field for the given differential equation at the 12 points indicated.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the $xy-$plane. Describe all points in the $xy-$plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$

1999 AB2 [An oldie but goodie!]



The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$ as shown above.

(a) Find the area of R

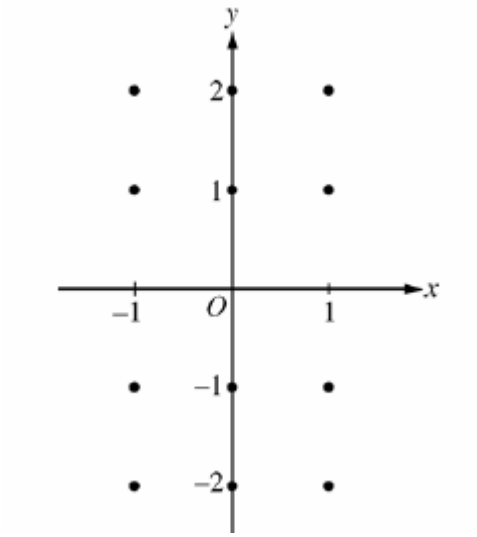
(b) Find the volume of the solid generated by revolving R about the $x-$axis

(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not evaluate, an equation involving an integral expression that can be used to find the value of k.

2005 AB 6 [non-calculator]

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -1$

2008 AB1B [calculator]

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$

(a) Find the area of R

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the $y-$axis are squares. Find the volume of this solid.