**FR1 [non-calculator]**

Let $f$ be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of $f$ is given by $f'(x) = \frac{1 - \ln x}{x^2}$

(a) Write the equation of the line tangent to the graph of $f$ at $x = e^2$

Need a slope and a point and the formula $y - y_1 = m(x - x_1)$

Point: $f(e^2) = \frac{\ln e^2}{e^2} = \frac{2}{e^2}$

Slope: $f'(e^2) = \frac{1 - \ln e^2}{(e^2)^2} = \frac{-1}{e^4}$

Equation of line tangent to the graph at $x = e^2$ is

$$y - \frac{2}{e^2} = \frac{-1}{e^4}\left(x - e^2\right)$$

(b) Find the x-coordinate of the critical point of $f$. Determine whether this point is a relative minimum, relative maximum, or neither for this function. Justify your answer [using Calculus].

$f'(x) = 0$ if $1 - \ln x = 0$

$$1 - \ln x = 0$$

$$1 = \ln x$$

$$e = x$$

On $(0, e)$ $f'(x) > 0$

On $(e, \infty)$ $f'(x) < 0$

At $x = e$, $f'(x)$ changes from positive to negative values. Hence, $f$ has a relative maximum value at $x = e$.

If you use the Second Derivative Test:
Since $f''(e) < 0$, then by the Second Derivative Test $f$ has a relative maximum value at $x = e$.

(c) Find $f''(x)$

$$f''(x) = \frac{(x^2)\left(\frac{-1}{x}\right) - (1 - \ln x)(2x)}{x^4}$$

This is an acceptable answer. If you simplify, then

$$f''(x) = \frac{2\ln x - 3}{x^3}$$

If you tried using the product rule, then

$$f''(x) = -2x^{-3} - \left[\frac{1 - 2\ln x}{x^3}\right]$$

**BONUS**

Find $\lim_{x \to 0^+} f(x) = -\infty$

**FR2 [Calculator**, so use your #$%!^&* calculator!]
A particle moves along the x-axis so that its velocity $v$ at any time $t$ [in seconds], for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} + \ln\left(t^2 + 1\right)$ in meters per second. The particle is at position $x = 5$ at time $t = 0$.

(a) Find the acceleration at time $t = 12$. Indicate units.

$a(t) = v'(t)$ so $a(12) = v'(12) \approx 0.742597 \frac{m}{\sec^2}$

(b) Find the position of the particle at time $t = 12$.

$$s(12) = s(0) + \int_0^{12} v(t)\, dt$$
$$\approx 5 + 66.9997548$$
$$\approx 71.9997548$$

(c) Find the total distance traveled by the particle over the interval $0 \le t \le 16$ seconds.

$$\text{Total distance traveled} = \int_0^{16} \left| v(t) \right| dt$$
$$\approx 101.9178126\ m$$

The only way I would have accepted $\int_0^{16} v(t)\, dt$ was if you explicitly stated that $v(t) \ge 0$ on [0, 16]

***BONUS***

Find the average speed of the particle over the interval $0 \le t \le 16$ seconds. Indicate units.

Speed = $\left| v(t) \right|$

Average speed = $\dfrac{1}{16-0} \int_0^{16} \left| v(t) \right| dt \approx 6.36986 \ \frac{m}{\text{sec}}$

***Most common mistakes made:***

Unsuccessful simplifications – why bother. [Read the directions!]

Wasting time trying to find derivatives and/or integrals in the calculator section.

Some people are still not writing coherent sentences for their justifications.

Some people do not appear to know the rules of $\ln x$ and/or $e^x$

Despite the fact that average value has appeared on numerous assessments, some people still do not know this formula.