AP PROBLEMS WITH $\ln x$ AND e^x
I have typed up the first and last pages of our handout because they are difficult to read. [I think that the inside pages are okay.]

2000 AB6 [***non-calculator***]

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$

(b) Find the domain and range of the function found in part (a)

2003 AB4B [***non-calculator- so leave your answers in terms of e***]

A particle moves along the $x-$axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$

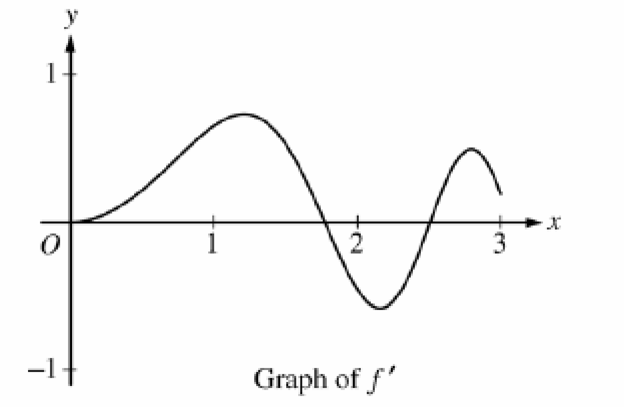
(a) Find the acceleration of the particle at time $t = 3$

(b) Is the speed of the particle increasing at time $t = 3$? Justify your answer [using Calculus!]

(c) Find all values of t at which the particle changes direction. Justify your answer [using Calculus!]

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

2006 AB2B [***calculator-friendly***]



Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{\frac{-x}{4}} \sin\left(x^2\right)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the open interval $1.7 < t < 1.9$. Justify.

(b) On the closed interval $0 \leq x \leq 3$, find the value of x at which f has an ***absolute maximum.*** Justify.

(c) Write an equation for line tangent to the graph of f at $x = 2$

2008AB2 [***calculator-friendly***]

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase ticket at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30p.m. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoid sum with three subintervals to estimate the ***average*** number of people waiting in line during the first **4** hours that tickets were on sale.

(c) For $0 \le t \le 9$ what is the fewest number of times at which $L'(t)$ must equal 0? Give a [Calculus] reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 500e^{\frac{-t}{2}}$ tickets per hour. Based on the model, how many tickets were sold by 3 p.m. $(t = 3)$, to the nearest number?

2008 AB2B [***calculator-friendly***]

For time $t \geq 0$, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by

$$g(x) = 0.05x\left(1 - e^{\frac{-x}{2}}\right)$$

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change ***with respect to time*** of the number of liters of gasoline used by the car when $t = 2$. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?