

**AP Problems with  $\ln x$  and  $e^x$  [Please do your work on a separate piece of paper]**

**2000 AB 6 non-calculator**

Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ .

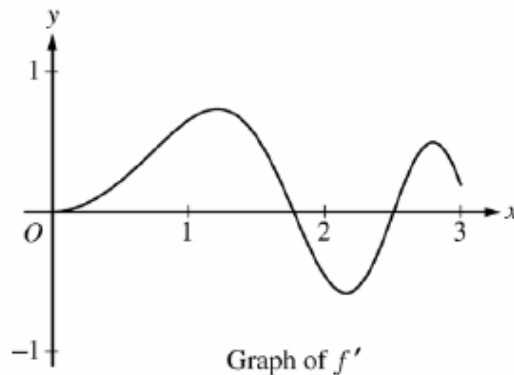
- (a) Find a solution  $y = f(x)$  to the differential equation satisfying  $f(0) = \frac{1}{2}$ .
- (b) Find the domain and range of the function  $f$  found in part (a).

**2003 AB 4B non-calculator**

A particle moves along the  $x$ -axis with velocity at time  $t \geq 0$  given by  $v(t) = -1 + e^{1-t}$ .

- (a) Find the acceleration of the particle at time  $t = 3$ .
- (b) Is the speed of the particle increasing at time  $t = 3$ ? Give a reason for your answer.
- (c) Find all values of  $t$  at which the particle changes direction. Justify your answer.
- (d) Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 3$ .

**2006 AB 2B calculator-friendly**



Let  $f$  be the function defined for  $x \geq 0$  with  $f(0) = 5$  and  $f'$ , the first derivative of  $f$ , given by  $f'(x) = e^{(-x/4)} \sin(x^2)$ . The graph of  $y = f'(x)$  is shown above.

- (a) Use the graph of  $f'$  to determine whether the graph of  $f$  is concave up, concave down, or neither on the interval  $1.7 < x < 1.9$ . Explain your reasoning.
- (b) On the interval  $0 \leq x \leq 3$ , find the value of  $x$  at which  $f$  has an absolute maximum. Justify your answer.
- (c) Write an equation for the line tangent to the graph of  $f$  at  $x = 2$ .

**From Barron's 2009 Practice Exam [which I paid for!]**

Non-calculator

Given the function  $f(x) = e^{2x}(x^2 - 2)$

- (a) Find all critical values of  $f(x)$
- (b) For what values of  $x$  is  $f$  decreasing?
- (c) Find the  $x$ -values of any extrema

**2007 AB 4 [non-calculator]**

A particle moves along the  $x$ -axis with position at time  $t$  given by  $x(t) = e^{-t} \sin t$  for  $0 \leq t \leq 2\pi$

- (a) Find the time  $t$  at which the particle is farthest to the left. Justify your answer.
- (b) Find the value of the constant  $A$  for which  $x(t)$  satisfies the equation  $Ax''(t) + x'(t) + x(t) = 0$  for  $0 < t < 2\pi$

**2007 AB 6 [non-calculator]**

Let  $f$  be the function defined by  $f(x) = k\sqrt{x} - \ln x$  for  $x > 0$ , where  $k$  is a positive constant

- (a) Find  $f'(x)$  and  $f''(x)$
- (b) For what value of the constant  $k$  does  $f$  have a critical point at  $x = 1$ ?  
For this value of  $k$ , determine whether  $f$  has a relative maximum, relative minimum, or neither at  $x = 1$ . Justify your answer.
- (c) For a certain value of the constant  $k$ , the graph of  $f$  has a point of inflection on the  $x$ -axis. Find this value of  $k$ .

**2005 AB3B [calculator]**

A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t$ , for  $0 \leq t \leq 5$ , is given by

$v(t) = \ln(t^2 - 3t + 3)$ . The particle is at position  $x = 8$  at time  $t = 0$ .

- (a) Find the acceleration of the particle at time  $t = 4$
- (b) Find all times in the open interval  $0 < t < 5$  at which the particle changes direction.  
During which time intervals, for  $0 \leq t \leq 5$  does the particle travel to the left?
- (c) Find the position of the particle at time  $t = 2$
- (d) Find the average speed of the particle over the interval  $0 \leq t \leq 2$

**From Be Prepared for the AP Calculus Exam [which I paid for!]**

*This is a calculator problem.*

Tourists visiting an island resort contracted a mystery illness over a 45 day period. The health authorities recorded the rate of new cases per day and some of the rates are listed in the table below.

$t$ day	$N(t)$ New cases per day
2	3
6	8
10	15
15	30
25	100
35	50
40	22
45	10

- (a) Assuming that the table above shows a sample of values for a continuous function  $N(t)$ ,

use a right-hand Riemann Sum with 8 subintervals to approximate  $\int_0^{45} N(t) dt$  and explain

the meaning.

- (b) After studying the spread of the disease, the health department authorities decided they

could model the number of new cases per day with  $R(t) = \frac{80000e^{-0.2t}}{\left(1 + 200e^{-0.2t}\right)^2}$  for

$0 \leq t \leq 50$  days. Use  $R(t)$  to find the **average** number of cases over the 45-day period.

- (c) The disease is considered eradicated when the number of new cases per day does not exceed five. Use  $R(t)$  to find on what day this will occur.

**2008 AB2 [calculator]****2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?

**2008 AB2 B [calculator]****2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

2. For time  $t \geq 0$  hours, let  $r(t) = 120(1 - e^{-10t^2})$  represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel  $x$  kilometers is modeled by  $g(x) = 0.05x(1 - e^{-x/2})$ .
- (a) How many kilometers does the car travel during the first 2 hours?
- (b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when  $t = 2$  hours. Indicate units of measure.
- (c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

