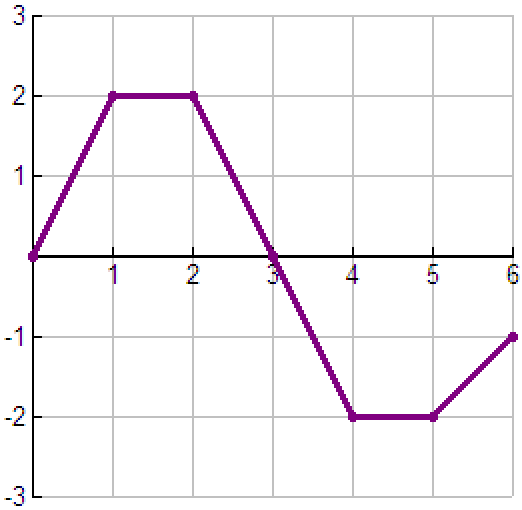
FR1



Graph of f

The graph of the function f shown above consists of five line segments. Let g be the function given by $g(x)=\int_0^x f(t)\,dt$

(A) Find $g(3)$, $g'(3)$ and $g''(3)$ [Show all work]

$$g(3)=\int_0^3 f(t)\,dt = (2)\frac{1+3}{2}=4$$

$$g'(x)=\frac{d}{dx}\int_0^x f(t)\,dt = f(x) \quad \text{So, } g'(3)=f(3)=0$$

$$g''(x)=f'(x) \quad \text{So, } g''(3)=f'(3)=\frac{f(4)-f(2)}{4-2}=-2$$

(B) Does g have a relative minimum, a relative maximum, or neither at $x=3$. [Justify]

$g'(x)=f(x)$ changes from positive to negative values at $x=3$. Hence, $g(x)$ has a relative maximum at $x=3$.

(C) Find any intervals where $g''(x) = 0$ [Justify]

One way: $g'(x) = f(x)$ is neither increasing nor decreasing on the intervals (1, 2) and (4, 5). Hence, $g''(x) = 0$ on these intervals.

Another way: $g''(x) = f'(x) = 0$ on the intervals (1, 2) and (4, 5) because on these intervals, the graph of f has horizontal segments.

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Free Response Directions

Show all your work. Be sure to write clearly and legibly using standard mathematical notation. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work will not receive credit. Justifications require that you clearly label functions, graphs, tables or other objects you use. Justifications also require that you use mathematical reasons. If you use a decimal approximation, then it should be accurate to three places after the decimal point.

FR2

The rate at which raw sewage [Eww!] enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at a constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$

(A) How many gallons of sewage entered the tank during the time interval $0 \le t \le 4$. Round your answer to the nearest gallon.

$$\int_0^4 E(t)\,dt \approx 3981.022458$$

Hence, about 3981 gallons of sewage entered the tank during $0 \le t \le 4$ hours.

(B) Define a function, $S(t)$ that will give the number of gallons at any time

One way: $S(t) = \int_0^t E(x)\,dx - \int_0^t 645\,dx$

Another way: $S(t) = \left[\int_0^t E(x)\,dx\right] - 645t$

Yet another way: $S(t) = \int_0^t \left[E(x) - 645\right] dx$

Note: Be careful about your variables and your grouping symbols.

(C) For $0 \le t \le 4$, at what time is the amount of sewage in the tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank?

$S'(t) = \frac{d}{dt}\int_0^t \left[E(x) - 645\right] dx = E(t) - 645$

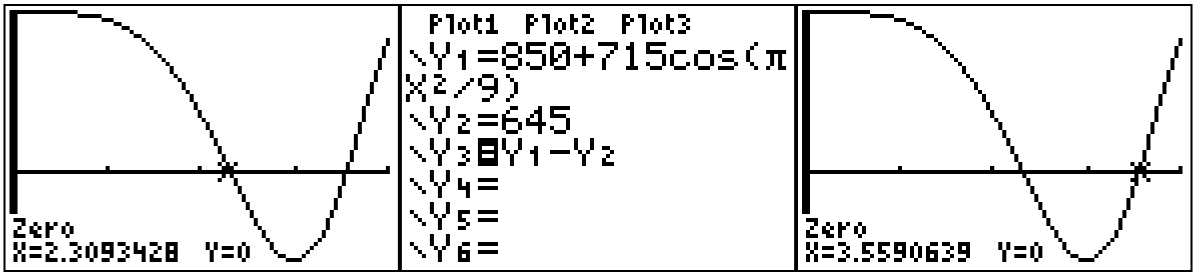
Let $S'(t) = 0$

$0 = E(t) - 645$

$S'(t) = 0$ at $t \approx 2.309$ and $t \approx 3.559$

At $t \approx 2.309$, the graph of $S'(t)$ changes from positive to negative values. So I will need to check $S(0)$, $S(4)$ and $S(2.309)$

[I used by trusty TI to find any critical values – which is what you should have done. Here is my TI-work but I would not include it in my solution for the AP.]



Note: Window is on [0, 4], then I "zoomfit".

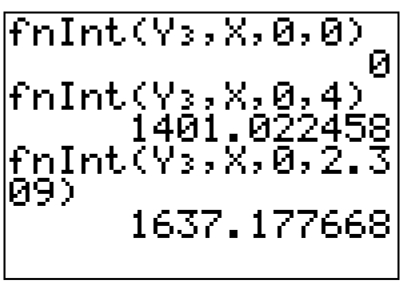
$$S(0) = 0$$

$$S(4) = \int_0^4 \left[E(x) - 645\right] dx \approx 1401 \textit{ gallons}$$

$$S(2.309) = \int_0^{2.309} \left[E(x) - 645\right] dx \approx 1637 \textit{ gallons}$$

Hence, the sewage amount is the greatest at $t \approx 2.309$ hours and the maximum amount is about 1637 gallons.

Note: I found the values by using my TI which you also should have done. Here is my TI-work [but I would not include in with my official AP work.]



See $Y3$ listing above.

FR3

The table below shows the depth of water, W , in the Newton River, as measured at 4-hour intervals during a day-long flood. Assume that W is a differentiable function of time t .

t (hour)	0	4	8	12	16	20	24
$W(t)$ In feet	32	36	38	37	35	33	32

(A) Find the approximate value of $W'(16)$ and indicate units of measure.

One way: $W'(16) \approx \dfrac{W(20)-W(16)}{20-16} = -\dfrac{1}{2}\,\dfrac{ft}{hr}$

Another way: $W'(16) \approx \dfrac{W(16)-W(12)}{16-12} = -\dfrac{1}{2}\,\dfrac{ft}{hr}$

(B) Estimate the average depth of the water, in feet, over the time interval $0 \le t \le 24$ hours by using a trapezoidal approximation with subintervals of length 4 hours.

Average depth = $\dfrac{1}{24-0}\displaystyle\int_0^{24} W(t)\, dt$

[I needed to see the above equation somewhere – in this form. Indefinite integrals are never considered for partial credit because their solutions would not be numerical.]

$$\int_0^{24} W(t)\, dt \approx \text{TRAP}$$

TRAP =

$$\frac{24-0}{2(6)}\left[W(0)+2W(4)+2W(8)+2W(12)+2W(16)+2W(20)+W(24)\right]$$

$= 2\,(422)$

$= 844$

Average depth = $\dfrac{1}{24-0}\int_0^{24} W(t)\;dt \;\approx \dfrac{844}{24} = 35.167\,ft$

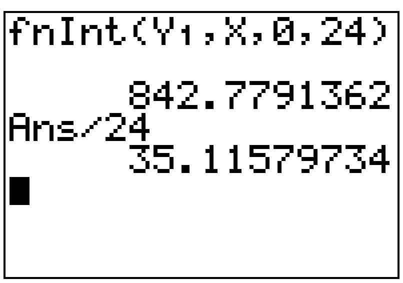
BONUS Free Response

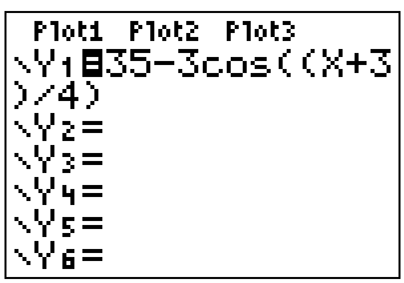
Mad scientists studying the flooding believe that can model the depth of the water with the function $F(t) = 35 - 3\cos\left(\dfrac{t+3}{4}\right)$, where $F(t)$ represents the depth of the water in feet after t hours. Use $F(t)$ to the average depth of the water over the time interval $0 \le t \le 24$ hours.

Average depth =

$$\frac{1}{24-0}\int_0^{24} F(t)\;dt \approx \frac{842.7791362}{24} = 35.11579734\;ft$$

Once again, I used by TI to find this value. Here is my TI-work.





Note: If you are finding a definite integral on the calculator-portion of any AP test or quiz, then you are expected to use your calculator.