My Chapter 4 Review 2009

1. $\int \frac{x^2 - x}{\sqrt{x}}\, dx$

$$= \int \left[x^{\frac{3}{2}} - x^{\frac{1}{2}} \right] dx$$

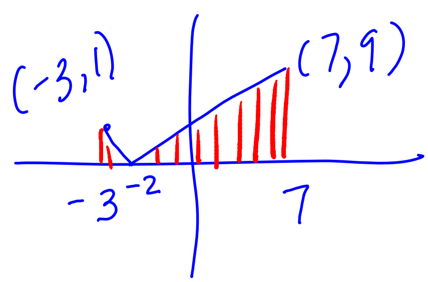
$$= \frac{2}{5} x^{\frac{5}{2}} - \frac{2}{3} x^{\frac{3}{2}} + C$$

2. If $f(x) = 2 + g(x)$ for $-1 \le x \le 1$, then what is the value of $\int_{-1}^{1} [f(x) - g(x)]\, dx$?

$$\int_{-1}^{1} \left[2 + \cancel{g(x)} - \cancel{g(x)} \right] dx$$

$$= \int_{-1}^{1} 2\, dx = 2x \Big|_{-1}^{1} = 2 - (-2) = 4$$

3. $\int_{-3}^{7} |\, x + 2 \,|\, dx$



$$= \frac{1}{2} + \frac{81}{2}$$

$$= 41$$

4. If $f(x) = \int_3^{\tan x} (1 - t^2)\,dt$, then $f'(x) = ?$

$$f'(x) = \frac{d}{dx} \int_3^{\tan x} (1 - t^2)\,dt$$

$$f'(x) = [1 - \tan^2 x]\left[\frac{d}{dx} \tan x\right]$$

$$f'(x) = \sec^2 x\,[1 - \tan^2 x]$$

5. $\int_0^x -\cos t\,dt =$

$$= -\sin t \Big|_0^x$$

$$= -\sin x$$

6. If $\int_1^c \frac{1}{x^2}\,dx = \frac{1}{2}$, then $c =$

$$\int_1^c x^{-2}\,dx = -\frac{1}{x}\Big|_1^c$$

$$\boxed{c = 2}$$

$$-\frac{1}{c} - \left(-\frac{1}{1}\right) = \frac{1}{2}$$

$$-\frac{1}{c} + 1 = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{c}$$

7. $\frac{d}{dx}\int_0^x -\cos t \; dt$

$= -\cos x$

8. $\int_0^5 (3x+1)^2 \; dx =$ $\int_0^5 [9x^2+6x+1]\, dx$

$= 3x^3+3x^2+x \Big|_0^5$

$= 375+75+5$

$= 455$

9. If $g(x)=\int_1^x \sec(2t+3)\,dt$, then $g'(x)=$

$g'(x) = \frac{d}{dx}\int_1^x \sec(2t+3)\,dt$

$g'(x) = \sec(2x+3)$

$y - 9 = -6(x-2)$ let $y=0$

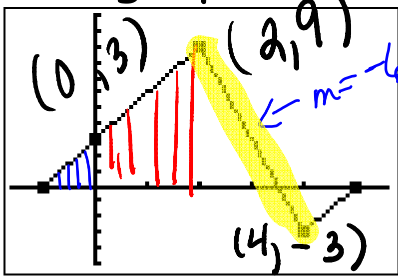
$-9 = -6x + 12$

$6x = 21$

$x = \frac{21}{6}$

FREE RESPONSE

The graph below is the graph of f on [-1, 5]



GRAPH of f

[GRAPH of $g'(x)$]

Let $g(x) = \int_0^x f(t)\,dt$

(A) Find $g(0)$, $g(-1)$ and $g(2)$

$g(0) = \int_0^0 f(t)\,dt = 0$

$g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^0 f(t)\,dt = -\frac{3}{2}$

$g(2) = \int_0^2 f(t)\,dt = \frac{3+9}{2}(2) = 12$

(B) Find $g'(x)$ and any critical value(s) of $g(x)$

$g'(x) = \frac{d}{dx}\int_0^x f(t)\,dt = f(x)$

$g'(x) = f(x) = 0$ at $x = -1, \frac{21}{6}, 5$

(C) Find any relative extrema of $g(x)$

at $x = \frac{21}{6}$ $g'(x) = f(x)$ changes from positive to negative values

Hence $g(x)$ has a rel max

at $x = \frac{21}{6}$

FR [calculator-friendly]

During a recent snowfall, several students monitored the accumulation of snow on the flat roof of their school. The table records the data they collected for the 12-hour period of the snowfall.

Number of hours (t)	**Rate of Snow [R(t)] (inches/hr)**
0	0
2	1.5
3	2.1
4.5	2.4
6.5	2.8
8	2.2
10.5	1.8
12	1.6

(A) Use a left-hand Riemann sum to approximate $\int_0^{12} R(t)\,dt$ and explain the real-life meaning.

$$LRAM = 2R(0) + 1R(2) + 1.5R(3) + 2R(4.5) + 1.5R(6.5) + 2.5R(8) + 1.5R(10.5)$$

$$= 2(0) + 1(1.5) + 1.5(2.1) + 2(2.4) + 1.5(2.8) + 2(2.2) + 1.5(1.8)$$

$$= 21.85 \text{ inches}$$

$\int_0^{12} R(t)\,dt$ is the number of inches that fell during $0 \le t \le 12$ hours

(B) Use your left-hand Riemann sum to estimate the average rate of snowfall in the 12-hour period. Remember to show your set-up!

we WANT AVERAGE VALUE

$$\frac{1}{12-0}\int_0^{12} R(t)\,dt$$

$$= \frac{21.85}{12} \ \frac{\text{inches}}{\text{hour}}$$

The problem above is a variation on a problem from the AMSCO AP Calculus preparation book [which I paid for!]

given $\int_0^5 f(x)\,dx = 4$ find $\int_0^5 [2-f(x)]\,dx$

$$\int_0^5 2\,dx - \int_0^5 f(x)\,dx$$

$$= 2x\Big|_0^5 - 4$$

$$= 10 - 4 = 6$$