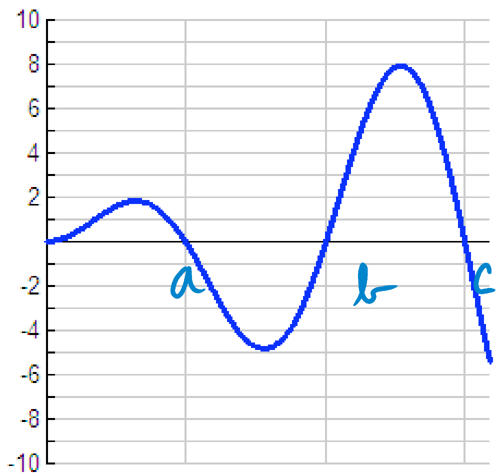
My Chapter 3 Take-Home Project [All work/steps must be shown]

I, ______________SKIPPY___________________________, claim that all work on this project is my own and that I have received NO help from anyone living, dead, or undead.

Skippy ____ [my signature]



graph of f'

1.

The graph of $f'(x)$, the derivative of a continuous function f, is shown above.

(a) On the closed interval $[a, b]$ where does the graph of f have critical values? [Explain fully]

f has critical values at $x = a$ and $x = b$ because $f'(a) = f'(b) = 0$ (3)

(b) On the open interval $(0, c)$ where does the graph of f have any relative extrema. [Explain fully]

At $x = a$, $f'(x)$ changes from positive to negative values. Thus, f has a relative maximum value at $x = a$ (3)

At $x = b$, $f'(x)$ changes from negative to positive values. Hence, f has a relative minimum value at $x = b$. (3)

2. Let $f(x)$ be the function defined by $f(x) = k + 12x + 3x^2 - 2x^3$, where k is a constant.

(a) On what interval is the function increasing? Justify your answer.

$f'(x) = 12 + 6x - 6x^2$ (3)

Let $f'(x) = 0$ So, $0 = 6x^2 - 6x - 12$

$0 = 6(x+1)(x-2)$ (1)

f has critical values at $x = -1$ and $x = 2$ (2)

On $(-\infty, -1)$ and on $(2, \infty)$ $f'(x) < 0$

Since $f'(x) > 0$ on $(-1, 2)$, then f is increasing on this interval. (2)

(b) If the relative maximum value of f is 4, then what is the value of k? [Justify]

At $x = 2$, $f'(x)$ changes from positive to negative values. Hence f has a relative maximum at $x = 2$. Let $f(2) = 4$

$4 = k + 24 + 12 - 16$ (4)

Hence, $k = -16$

(c) Find the interval where the function f is concave up. [Justify]

$f''(x) = 6 - 12x$ (2)

$f''(x) = 0$ when $x = \frac{1}{2}$ (1) On $(0.5, \infty)$ $f''(x) < 0$

On $(-\infty, 0.5)$, $f''(x) > 0$. Hence f is concave up on this interval. (2)

3. Let g be a twice-differentiable function such that $g(3)=7$ and $g(7)=3$.

Let h be the function given by $h(x)=g(g(x))$.

(a) Explain why there must be a value c, $3<c<7$, such that $h'(c)=1$

Preliminary work:

$$h(3)=g(g(3)) \qquad h(7)=g(g(7)) \quad (1)$$
$$=g(7) \quad (1) \qquad = g(3)$$
$$= 3 \qquad = 7$$

By the Mean Value Theorem, there is a c, $3<c<7$, such that $h'(c)=\dfrac{h(7)-h(3)}{7-3}$. (2)

Since, $\dfrac{h(7)-h(3)}{7-3} = \overset{(1)}{\dfrac{7-3}{7-3}}$, then there is a c, $3<c<7$, such that $h'(c)=1$
$=1$

(b) Show that $h'(3)=h'(7)$. Use this result to explain why there must be a value k, $3<k<7$, such that $h''(k)=0$

Preliminary work: $h'(x)=[g'(g(x))]\,[g'(x)]$

$$h'(3)=[g'(g(3))]\,[g'(3)] \qquad h'(7)=[g'(g(7))]\,[g'(7)]$$
$$= g'(7)\ g'(3) \quad (1) \qquad = g'(3)\ g'(7) \quad (1)$$

Holy smokes! They are equal!!!

By the Mean Value Theorem [really?! Again!!], there is a k, $3<k<7$, such that $h''(k)=\dfrac{h'(7)-h'(3)}{7-3}$. Since $h'(3)=h'(7)$, then there must be a k, $3<k<7$, such that $h''(k)=0$.

(2)

4. It took 20 seconds for the temperature to rise from $0^\circ F$ to $212^\circ F$ when a thermometer was take from a freezer and placed in boiling water. Explain [using Calculus] why at some moment in that interval the mercury in the thermometer was rising at exactly $10.6^\circ F/\sec$.

We may assume that the temperature is a continuous and differential function of time.

Let $m(t)$ be the measure of the mercury in the thermometer at time, t

$$m(0)=0^\circ,\ \ m(20)=212^\circ$$

By the Mean Value Theorem there is a time t, $0<t<20$, such that $m'(t)=\dfrac{m(20)-m(0)}{20-0}$

$=$

$$\frac{212-0}{20-0}=\frac{10.6^\circ F}{\sec}$$

(4)

Hence, there is a time t, 0 < t < 20, such the mercury in the thermometer was rising at exactly $10.6^\circ F/\sec$

5. A particle moves along a horizontal line with positive velocity $v(t)$ where v is a differentiable function of time t. The velocity of the particle at selected times is given in the table below.

t (sec)	0	2	4	6	8	~~10~~	12
$v(t)$ (cm/sec)	37	17	5	1	6	17	38

(a) Find the average acceleration of the particle over the time interval $8 \le t \le 10$ seconds.

Average acceleration for this interval = $\dfrac{v(10)-v(8)}{10-8}$ (2)

av acc

$= \dfrac{17-6}{10-8}$ (1)

$= \dfrac{11}{2}\ \dfrac{cm}{\sec^2}$ (2)

(b) Estimate a value for $a(5)$. Show correct units.

$a(t) = v'(t)$ so, $a(5) = v'(5) \approx \dfrac{v(6)-v(4)}{6-4}$ (2)

$= \dfrac{1-5}{6-4}$ (1)

$= -2\ \dfrac{cm}{\sec^2}$ (2)

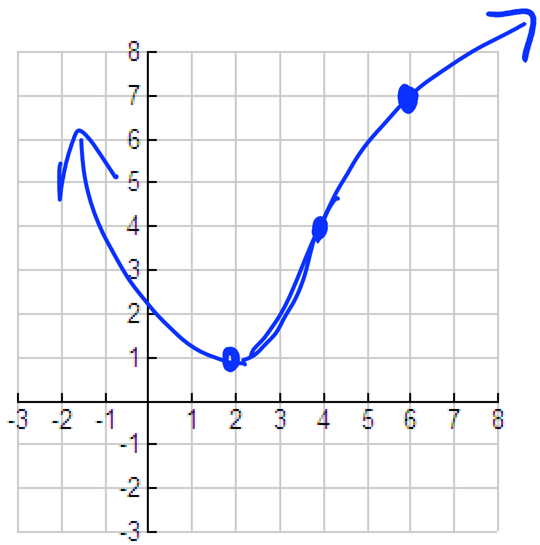
(c) Based on the values in the table, what is the smallest number of times at which the acceleration of the particle could equal zero on the open interval $t, 0 < t < 12$ seconds.

By the Mean Value Theorem, there must be a $t,\ 2 < t < 10$, such that

$v'(t) = a(t) = \dfrac{v(10) - v(2)}{10 - 2}$. Since $v(10) = v(2) = 17 cm/\sec$, then there must be a a $t,\ 2 < t < 10$, such that $a(t) = 0$. Since $2 < t < 10$ is contained within 0 < t < 12, then there is at least one time that $a(t) = 0$ on 0 < t < 12 seconds. (3)

6. Sketch a continuous curve $y = f(x)$ with the properties listed below. Label coordinates where possible.

x	$f(x)$	Properties	
$x < 2$		$f' < 0,\ f'' > 0$	(2)
2	1	$f' = 0$	(2)
$2 < x < 4$		$f' > 0,\ f'' > 0$	(2)
4	4	$f'' = 0$	(2)
$4 < x < 6$		$f' > 0,\ f'' < 0$	(2)
6	7	$f' = 0$	(2)
$x > 6$		$f' < 0, f'' < 0$	(2)



8
7
6
5
4
3
2
1
-1
-2
-3
-3 -2 -1 1 2 3 4 5 6 7 8