

Stuff to remember:

Critical values/numbers are where $f'(x)$ is either equal to zero OR $f'(x)$ is undefined [but $f(x)$ is defined]

Possible points of inflection are where $f''(x)$ is either equal to zero OR $f''(x)$ is undefined [but $f(x)$ is defined]

If a function is differentiable at a point, then the function is continuous at that point.

How to justify:

[Assume that the function is continuous and differentiable]

If $f'(x) > 0$ for every x in (a, b) , then $f(x)$ is increasing on (a, b)

If $f'(x) < 0$ for every x in (a, b) , then $f(x)$ is decreasing on (a, b)

If $f''(x) > 0$ for every x in (a, b) , then $f(x)$ is concave up AND $f'(x)$ is increasing on (a, b)

If $f''(x) < 0$ for every x in (a, b) , then $f(x)$ is concave down AND $f'(x)$ is decreasing on (a, b)

If $f'(x)$ is increasing on (a, b) , then $f(x)$ is concave up on (a, b) AND $f''(x) > 0$ on (a, b)

If $f'(x)$ is decreasing on (a, b) , then $f(x)$ is concave down on (a, b) AND $f''(x) < 0$ on (a, b)

Let $x = a$ be a critical value for $f(x)$ [in other words, $f'(a) = 0$ OR $f'(a)$ is undefined but $f(a)$ is defined]

To show that $(a, f(a))$ is a relative/local minimum:

"At $x = a$, $f'(x)$ changes from negative to positive values. Hence, f has a relative minimum at $x = a$."

To show that $(a, f(a))$ is a relative/local maximum:

"At $x = a$, $f'(x)$ changes from positive to negative values. Hence, f has a relative maximum at $x = a$."

Let $f''(b) = 0$ or $f''(b)$ be undefined but $f(b)$ is defined

To show that $(b, f(b))$ is a point of inflection you must justify with one of the following statements:

"At $x = b$, $f''(b)$ changes from positive to negative values. Hence, f has a point of inflection at $x = b$."

OR

"At $x = b$, $f''(b)$ changes from negative to positive values. Hence, f has a point of inflection at $x = b$."

To justify an absolute maximum or absolute minimum

The candidates are the critical values of $f(x)$ AND the endpoints! First justify any relative min/max AND find their function values. Then compare the function values to the endpoint values and decide which is the absolute min/max.

Extreme Value Theorem

If a function, f , is continuous on $[a, b]$, then f has both a maximum and a minimum value on $[a, b]$.

Mean Value Theorem

Your justification statement should look like:

By the Mean Value Theorem there is a c , $a < c < b$, such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

You need to supply all of the values!

Rolle's Theorem [A "special" case of MVT]

Your justification statement should look like:

By Rolle's Theorem, since f is continuous on $[a, b]$ and differentiable on (a, b) AND $f(a) = f(b)$, then there is a c , $a < c < b$, such that $f'(c) = 0$.

Note: You can always just use MVT.

Horizontal TANGENTS occur at $x = a$ if $f'(a) = 0$

Vertical TANGENTS occur at $x = a$ if $f'(a)$ is undefined but $f(a)$ is defined

If $f'(x)$ is of the form $f'(x) = \frac{g(x)}{h(x)}$, then f will have a horizontal tangent when $g(x) = 0$ AND f will have a vertical tangent if $h(x) = 0$. [Of course $f(x)$ must be defined for those values]

Horizontal Asymptote

$y = b$ is a horizontal asymptote of the graph of $y = f(x)$ if either

$$\lim_{x \rightarrow \infty} f(x) = b \text{ OR } \lim_{x \rightarrow -\infty} f(x) = b$$

Limits of Rational Functions as $x \rightarrow \pm\infty$ [End Behavior]

$$\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)} = 0 \text{ if the degree of } f(x) < \text{ the degree of } g(x)$$

$$\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)} = \infty \text{ or "dne" if the degree of } f(x) > \text{ the degree of } g(x)$$

$$\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)} \text{ is finite [there is a horizontal asymptote] if}$$

$$\text{degree of } f(x) = \text{degree of } g(x)$$

Remember: A graph is NOT justification. You must use Calculus to justify any extrema, points of inflection, horizontal asymptotes, etc.

Remember:

If $s(t)$ is the position function, then

Velocity, $v(t) = s'(t)$

Acceleration, $a(t) = v'(t) = s''(t)$

Speed is $|v(t)|$

The speed of a particle is increasing at $x = c$ if $v(c)$ and $a(c)$ are either both positive or both negative.

The speed of a particle is decreasing at $x = c$ if $v(c)$ and $a(c)$ have different signs.

To justify that the speed of a particle is increasing at $x = c$ you need to clearly state that $v(c) > 0$ AND $a(c) > 0$
OR $v(c) < 0$ AND $a(c) < 0$ in a statement.

To justify that the speed of a particle is decreasing at $x = c$ you need to clearly state that $v(c) > 0$ AND $a(c) < 0$
OR $v(c) < 0$ AND $a(c) > 0$ in a statement.

OPTIMIZATION

If necessary, you might want to draw a picture of what the problem is about.

Figure out what you are trying to optimize.

Find a primary equation that fits what you are trying to optimize.

If necessary, find a secondary equation that will allow you to re-write your primary equation in terms of just ONE VARIABLE.

Now just find the min or max using standard Calculus min/max techniques.