My Chapter 3 AP Calculus AB Test Skippy

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Part I – A calculator may NOT be used on this portion of the test

Multiple-Choice Directions

Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding bubble on your Scantron sheet. Do not spend too much time on any one problem. No credit will be given for anything written in your scratch work. Please be sure that your name is on this page AND on your Scantron form.

1. $\lim_{x\to\infty} \frac{3x^3-2x+1}{1+2x-x^3}$ is

(A) -3
(B) $\frac{-1}{2}$
(C) $\frac{-1}{3}$
(D) 3
(E) 1

$$\lim_{x\to\infty} \frac{3x^3}{-x^3} = -3$$

2. $\lim_{x\to+\infty} \frac{x-\frac{1}{2x}}{2x+\frac{1}{6x}}$ is

(A) -3
(B) $\frac{-1}{2}$
(C) $\frac{-1}{3}$
(D) $\frac{1}{2}$
(E) 2

$$\lim_{x\to\infty} \frac{x}{2x} = \frac{1}{2}$$

3. $\lim_{x\to-\infty} \frac{x^4+2x^2+4}{5-3x-x^3}$ is

(A) -1
(B) 1
(C) $\frac{1}{5}$
(D) $\frac{-1}{5}$
(E) Does not exist

$$\lim_{x\to-\infty} \frac{x^4}{-x^3} = \lim_{x\to-\infty} -x = \text{dne}$$

4. If f is the function defined by $f(x)=\frac{5x^7}{7}+4x^6+6x^5+x+1$, what are all the $x-$coordinates of the points of inflection of the graph of f?

(A) -2 only
(B) 0 only
(C) 2 only
(D) -2 and 0 only
(E) -2, 0, and 2

$f'(x) = 5x^6 + 24x^5 + 30x^4 + 1$
$f''(x) = 30x^5 + 120x^4 + 120x^3$
$0 = 30x^3(x^2+4x+4)$
$0 = 30x^3(x+2)^2$

Possible PofI at $x=0$, $x=-2$

$(-\infty,-2)$ $f''<0$ $(-2,0)$ $f''<0$ $(0,\infty)$ $f''>0$

5. Suppose that $f(x)$ is a twice-differentiable function on the closed interval $[a,b]$. If there is a number c, $a<c<b$, for which $f'(c)=0$, which of the following must be true?

I. $f(a)=f(b)$
II. f has a relative extremum at $x=c$ NOT ALWAYS
III. f has a point of inflection at $x=c$ MAYBE

$\frac{f(a)-f(b)}{a-b}=0$

(A) None of these are true
(B) I only
(C) II only
(D) I and II
(E) I, II, and III

6. For what value of k will $y=\frac{8x+k}{x^2}$ have a relative maximum at $x=4$?

$y'(4)=0$

(A) -32
(B) -16
(C) 0
(D) 16
(E) 32

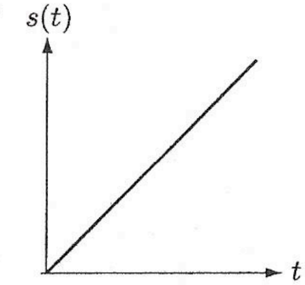
$y = 8x^{-1} + kx^{-2}$
$y' = -8x^{-2} - 2kx^{-3}$ let $y'=0$
$0 = -\frac{8}{x^2} - \frac{2k}{x^3}$ let $x=4$
$0 = -\frac{8}{16} - \frac{2k}{64}$
$k=-16$

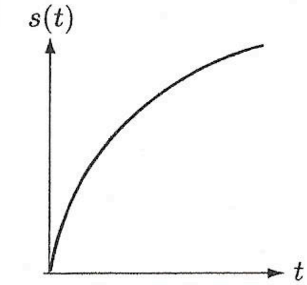
7. What are all values of x for which the graph of $y = x^3 - 6x^2$ is concave down?

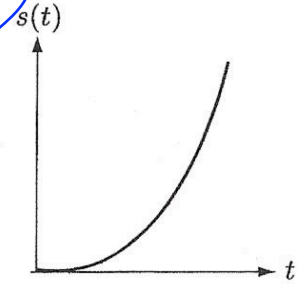
(A) $0 < x < 4$
(B) $x > 2$
(C) $x < 2$
(D) $x < 0$
(E) $x > 4$

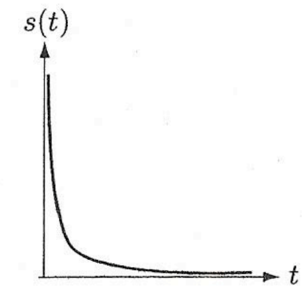
$y' = 3x^2 - 12x$
$y'' = 6x - 12$
$y'' = 0$ IF $x = 2$
$(-\infty, 2)$ $y'' < 0$
$(2, \infty)$ $y'' > 0$

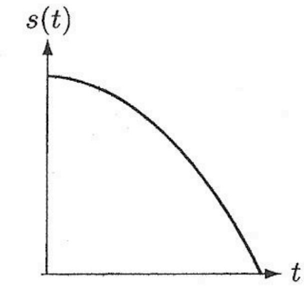
8. Which graph best represents the graph of the position of a particle, $s(t)$, if the particle's velocity and acceleration are both positive?

(A)


(B)


(C)


(D)


(E)


IF $v(t) > 0$ then $s(t)$ is increasing
IF $a(t) > 0$ then $s(t)$ is concave up

Directions for Free Response:
Show ALL work/steps. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as on your solutions. Answers without supporting work will not receive credit. Justifications require mathematical reasoning. Your work must be expressed in standard mathematical notation. Be sure to write clearly and legibly.

FR1

QuickTime™ and a
decompressor
are needed to see this picture.

The figure above shows the graph of the derivative of a continuous function f for $-4 \le x \le 4$. The graph of f' has x–intercepts at $x = -2$ and $x = 1.5$

(a) For what values of x is $f(x)$ increasing. Justify your answer.

(b) For what value(s) of x does f have any relative extrema? Justify your answer.

(c) Find all values of x where f has points of inflection. Justify your answer.

FR2

A function $f(x)$ is ***continuous*** on the closed interval $[-3, 3]$ and $f(-3) = 1$,
$f(-1) = 7$, $f(1) = 5$ and $f(3) = 4$.

The functions $f'(x)$ and $f''(x)$ have the properties given in the table below.

x	$-3 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$
$f'(x)$	Positive	Does not exist	Negative	0	Negative
$f''(x)$	Positive	Does not exist	Positive	0	Negative

(A) What are the $x-$coordinates of all extrema of f on the interval $(-3, 3)$. State which are min and max and justify your answer.

(B) What are the x-coordinates of all points of inflection of f on the interval $(-3, 3)$. Justify your answer.

(C) On the grid below, draw a possible graph of f that satisfies all of the given properties of f.

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Part II – A calculator may be necessary for some problems on this portion of the test

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51. Let f be a function that is everywhere differentiable. The value of f' is given for several values of x in the table below.

x	-10	-5	0	5	10
f'	-2	-1	0	1	2

at $x=0$ f' changes from neg to pos

f' always increasing mean $f''>0$ and f concave up

If f' is always increasing, which statement about $f(x)$ must be true?

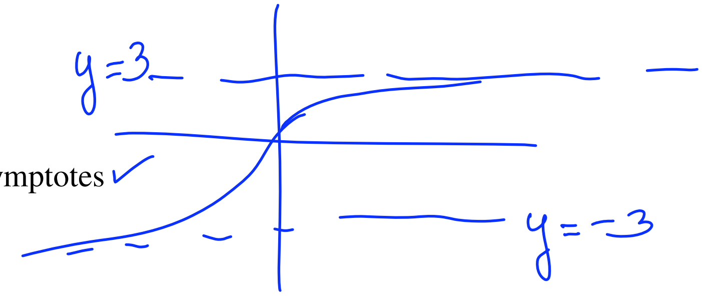
(A) $f(x)$ has a relative minimum at $x=0$
(B) $f(x)$ is concave down for all x
(C) $f(x)$ has a point of inflection at $(0, f(0))$
(D) $f(x)$ passes through the origin
(E) $f(x)$ is an odd function

52. If f is a function such that $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=0$, which of the following must be true?

(A) $\lim_{x\to a} f(x)$ does not exist
(B) $f(a)$ does not exist
(C) $f'(a)=0$
(D) $f(a)=0$
(E) $f(x)$ is continuous at $x=0$

$f'(a)=0$

53. If f is a continuous odd function and $\lim\limits_{x \to -\infty} f(x) = -3$, which of the following statements must be true?



I. $\lim\limits_{x \to +\infty} f(x) = 3$

II. There are no vertical asymptotes

III. The lines $y = 3$ and $y = -3$ are horizontal asymptotes

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

54. A particle moves along the x- axis so that its position at any time $t > 0$ is given by $s(t) = t^3 + 22t - 6\cos(\pi t)$. For what value of t is the velocity negative?

(A) $t = \frac{1}{2}$
(B) $t = 1$
(C) $t = \frac{3}{2}$
(D) $t = 2$
(E) The velocity is never negative

graph on TI

let $y_1 = s(t)$
$y_2 = \text{nDer}(y_1, x, x)$
look only at y_2

BONUS Multiple choice [Please mark as #55 on your Scantron}
Consider the function $g(x) = \tan(x+2)$ in the open interval $-4 < x < 5$. How many times are the tangents to $g(x)$ parallel to the line $y = 2x - 1$?

(A) never
(B) 2
(C) 4
(D) 5
(E) an infinite number of times

$y = 2x - 1$ has a slope of 2

let $g'(x) = 2$
use your TI
count the intersections

Directions for Free Response:
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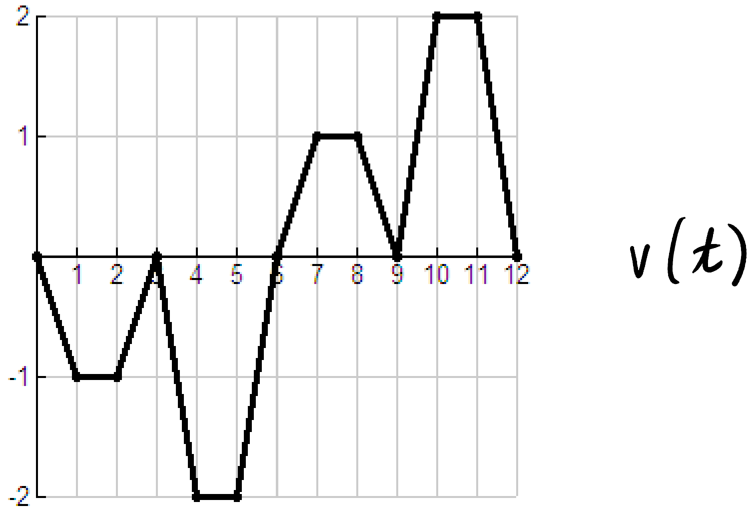
FR3



Graph of $v(t)$

A particle moves along a horizontal line such that its position is given by $s(t)$, which is a differentiable function on the closed interval [0, 12]. The graph of the velocity, $v(t)$, of the particle is shown above. The position of the particle at $t = 4$ is 10.

(a) Write the equation of the tangent line to the graph of $s(t)$ at $t = 4$.

(b) When is the particle at rest during the open interval $0 < t < 12$? Justify your answer.

(c) Find any time(s), t, when the particle changes direction. Justify your answer.

(d) Find the acceleration of the particle when $t = 9.5$. Justify your answer.

FR4

You are planning to make an open rectangular box from an 8 inch by 15 inch piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are dimensions of the box of largest volume you can make this way AND what is its volume? [Indicate units AND clearly state your domain of your primary equation.]

FREE REPONSE BONUS [OPTIONAL]

Let f be a differentiable function such at $f(0)=11$ and $f'(0)=17$. Use the tangent line to the graph of f at $x=0$ to approximate a value for $f(0.05)$.

If the graph of $f''(x)>0$ for all values of x, then is your estimate an over-estimate or an under-estimate? [Justify]