

$$\begin{aligned}
 1. \quad \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} &= \left. \frac{d}{dx} x^3 \right|_{x=2} \\
 &= \left. 3x^2 \right|_{x=2} \\
 &= 12
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - \tan\left(\frac{\pi}{4}\right)}{h} &= \left. \frac{d}{dx} \tan x \right|_{x=\frac{\pi}{4}} \\
 &= \left. \sec^2 x \right|_{x=\frac{\pi}{4}} \\
 &= 2
 \end{aligned}$$

3. Write the equation of the line tangent to the graph of $y = 5x - \sin x$ at $x = 2\pi$

$$y(2\pi) = 10\pi \quad \text{Point } (2\pi, 10\pi)$$

$$y'(x) = 5 - \cos x$$

$$y'(2\pi) = 5 - 1 = 4$$

equation of tangent line $y - 10\pi = 4(x - 2\pi)$

4. Let $f(x) = \begin{cases} 9x - 4, & x \leq 1 \\ 4x^2 + 1, & x > 1 \end{cases}$

$$f'(x) = \begin{cases} 9, & x < 1 \\ 8x, & x > 1 \end{cases}$$

Is this function continuous and/or differentiable at $x = 1$

To be continuous
 $f(1) = 5$

$$\lim_{x \rightarrow 1^-} f(x) = 5 \quad \lim_{x \rightarrow 1^+} f(x) = 5$$

$$\lim_{x \rightarrow 1} f(x) = 5 = f(1) \quad \text{Hence, CONTINUOUS at } x=1$$

$$\lim_{x \rightarrow 1^-} f'(x) = 9$$

$$\lim_{x \rightarrow 1^+} f'(x) = 8$$

Hence, NOT DIFFERENTIABLE AT $x=1$

5. If $\lim_{h \rightarrow 0} \frac{f(10+h) - f(10)}{h} = 25$, then which of the following may we assume to be true:

$$f'(10) = 25$$

- I $f(10) = 25$
- II $f'(10) = 25$
- III f is both continuous and differentiable at $x = 10$

6. Given the following information about differentiable functions $f(x)$ and $g(x)$ at $x = 2$ determine the following:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	2π	e

(a) $\frac{d}{dx} [f(g(x))] \text{ at } x = 2$

$$\begin{aligned}
 &= f'(g(x)) g'(x) \Big|_{x=2} \\
 &= f'(g(2)) g'(2) \\
 &= f'(2) g'(2) = 2\pi e
 \end{aligned}$$

(b) $\frac{d}{dx} \left(\frac{1}{f(x)} \right) \text{ at } x = 2$

$$\begin{aligned}
 &= \frac{0 - f'(x)}{[f(x)]^2} \Big|_{x=2} \\
 &= \frac{-f'(2)}{[f(2)]^2} = \frac{-2\pi}{64}
 \end{aligned}$$

(c) $\frac{d}{dx} \left[\frac{g(x)}{f(x)} \right] \text{ at } x = 2$

$$\begin{aligned}
 &= \frac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2} \Big|_{x=2} \\
 &= \frac{8e - (2)(2\pi)}{64}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \text{Find } \frac{d}{dx} [\sin \sqrt[3]{x}] &= \frac{d}{dx} \sin u \\
 &= \cos u \frac{du}{dx} \\
 &= [\cos \sqrt[3]{x}] \left[\frac{1}{3} x^{-\frac{2}{3}} \right]
 \end{aligned}$$

$$\begin{aligned}
 u &= \sqrt[3]{x} \\
 \frac{du}{dx} &= \frac{1}{3} x^{-\frac{2}{3}}
 \end{aligned}$$

8. Find $f'(x)$ if $f(x) = \frac{2x+3}{3x+2}$

$$\begin{aligned}
 f'(x) &= \frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2} \\
 &= \frac{-5}{(3x+2)^2}
 \end{aligned}$$

9. Given $25x^2 + 8x - 16y^2 - 4y - 9 = 0$

(a) Find $\frac{dy}{dx}$

$$\frac{d}{dx} 25x^2 + \frac{d}{dx} 8x - \frac{d}{dx} 16y^2 - \frac{d}{dx} 4y - \frac{d}{dx} 9 = \frac{d}{dx} 0$$

$$50x + 8 - 32y \frac{dy}{dx} - 4 \frac{dy}{dx} = 0$$

$$50x + 8 = \frac{dy}{dx} (32y + 4)$$

$$\frac{50x + 8}{32y + 4} = \frac{dy}{dx}$$

(b) Find any value(s) of x where the curve has a horizontal tangent

Horizontal tangent if $\frac{dy}{dx} = 0$, so let $50x + 8 = 0$. Hence, at $x = \frac{-8}{50}$

(c) Find the value(s) of y where the curve has a vertical tangent

Vertical tangent if $\frac{dy}{dx}$ is undefined, so let $32y + 4 = 0$. Hence, at $y = \frac{-4}{32}$

10. What is the slope of the tangent line to the graph of $y = \tan(2x)$ at $x = \frac{\pi}{8}$?

$$y = \tan u$$

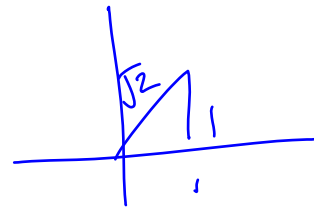
$$y' = \sec^2 u \frac{du}{dx}$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$y' = 2 \sec^2(2x)$$

$$y'(\frac{\pi}{8}) = 2 \sec^2(\frac{\pi}{4}) = 4$$



Some calculator-friendly problems:

11. A particle moves along the x-axis so that any time $t \geq 0$, its velocity is given by $v(t) = t^3 \sin t$. Find the acceleration at $t = 3$ AND determine if the speed is increasing or decreasing at $t = 3$

$$a(t) = v'(t) \text{ so } a(3) = v'(3)$$

Plot1	Plot2	Plot3	X	Y1	Y2
$Y_1 = (X^3) * (\sin(X))$			3	3.8102	3.8102
$Y_2 = \text{fnDeriv}(Y_1, X,$					
$X)$					
$Y_3 =$					
$Y_4 =$					
$Y_5 =$					
					$Y_2 = -22.91956341$

$$a(3) \approx -22.91956$$

$$v(3) \approx 3.8102$$

At $t = 3$ the speed is decreasing because $v(3) > 0$ and $a(3) < 0$

12. A pebble is thrown into a pond forming ripples whose radius increases at a rate of 4 inches/second. How fast is the area of the ripple changing when the radius is 12 inches?

$$A = \pi r^2$$

$$\frac{d}{dt} A = \frac{d}{dt} \pi r^2$$

$$\frac{dr}{dt} = 4 \frac{\text{in.}}{\text{sec}}$$

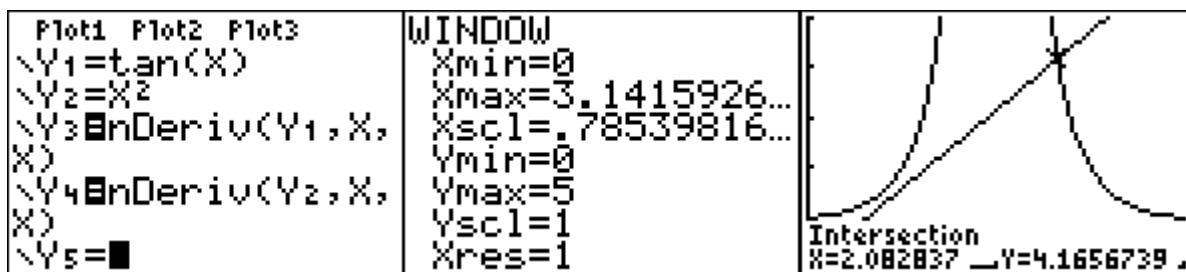
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$r = 12$$

$$\frac{dA}{dt} = 2\pi (12 \text{ inches}) (4 \frac{\text{in}}{\text{sec}}) = 96\pi \frac{\text{in}^2}{\text{sec}}$$

13. On the interval $[0, \pi]$, where do the graphs of $f(x) = \tan x$ and $g(x) = x^2$ have parallel tangent lines?

IF PARALLEL, then same slope
 so find out where $f'(x) = g'(x)$ on $[0, \pi]$



Hence, at $x \approx 2.082837$

- 14.

The position $s(t)$ of a particle is measured every 10 seconds and is provided in the table below.

t [in seconds]	0	10	20	30	40	50
$s(t)$ [in feet]	0	12	15	17	30	60

- (A) Estimate the instantaneous velocity of the particle at $t = 25$ seconds. Include units.

$$v(25) \approx \frac{s(30) - s(20)}{30 - 20}$$

$$= \frac{17 - 15}{10} = \frac{1}{5} \frac{\text{ft}}{\text{sec}}$$

- (B) Find the average velocity for the time interval $[0, 50]$

$$\text{AV VEL ON } [0, 50] = \frac{s(50) - s(0)}{50 - 0}$$

$$= \frac{60 - 0}{50} = \frac{6}{5} \frac{\text{ft}}{\text{sec}}$$

15. Find the following:

(A) $\frac{d}{dx}[7\cos^3(\pi x)]$

$$= 7 \frac{d}{dx} [\cos(\pi x)]^3$$

$$= 7 \frac{d}{dx} [u^3]$$

$$= 21u^2 \frac{du}{dx}$$

$$= -21\pi [\cos(\pi x)]^2 [\sin(\pi x)]$$

$$u = \cos \pi x$$
$$\frac{du}{dx} = -\pi \sin \pi x$$

(B) $\frac{d}{dt} \left[\frac{\pi}{3} r^2 h \right]$

$$= \frac{\pi}{3} \left[\left(2r \frac{dr}{dt} \right) (h) + \left(\frac{dh}{dt} \right) (r^2) \right]$$

$$= \frac{2\pi}{3} r h \frac{dr}{dt} + \frac{\pi}{3} r^2 \frac{dh}{dt}$$

16.

2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

6. Consider the closed curve in the xy -plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x -axis? Explain your reasoning.

(a) $2x + 2 + 4y^3 \frac{dy}{dx} + 4 \frac{dy}{dx} = 0$

$$(4y^3 + 4) \frac{dy}{dx} = -2x - 2$$

$$\frac{dy}{dx} = \frac{-2(x+1)}{4(y^3+1)} = \frac{-(x+1)}{2(y^3+1)}$$

2 : $\begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\left. \frac{dy}{dx} \right|_{(-2,1)} = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4}$

Tangent line: $y = 1 + \frac{1}{4}(x + 2)$

2 : $\begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

3 : $\begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the} \\ \quad \text{equation of the curve} \\ 1 : \text{answer} \end{cases}$

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

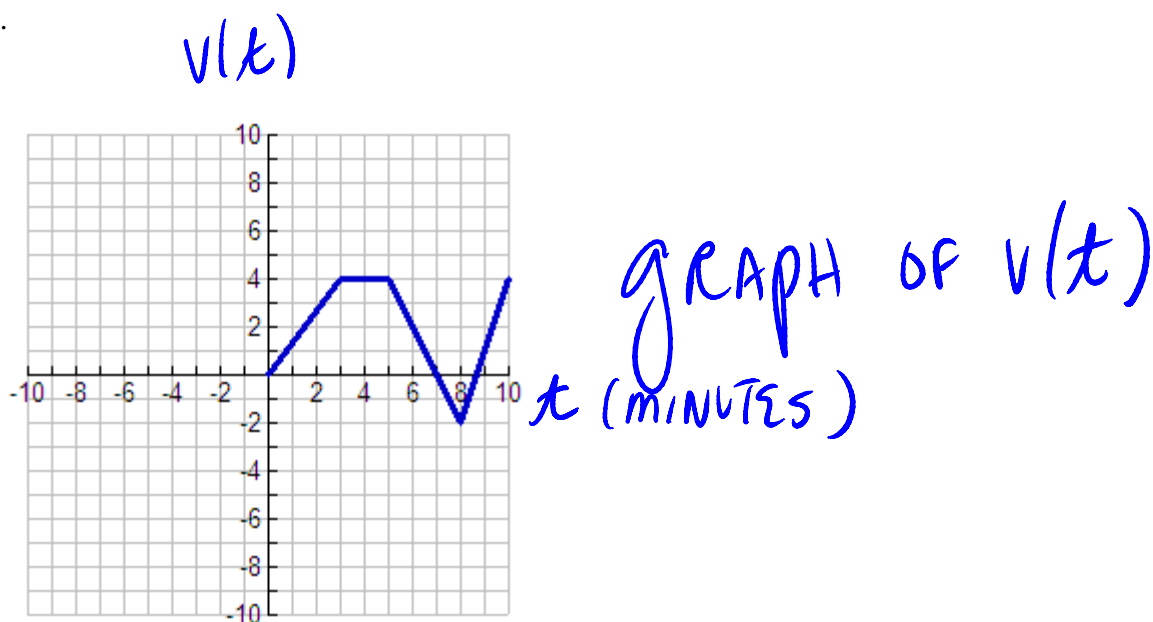
The curve crosses the x -axis where $y = 0$.

$$(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$$

No, the curve cannot have a horizontal tangent where it crosses the x -axis.

2 : $\begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$

17.



The graph above is the velocity graph of a particle moving along the x-axis.

(A) When is the particle at rest? Justify.

The particle is at rest when $v(t) = 0$. Hence, at $t = 0$, $t = 7$, and $t \approx 8.5$, the particle is at rest.

(B) When does the particle change direction? Justify.

At $t = 7$, $v(t)$ changes from positive to negative values and at $t \approx 8.5$, $v(t)$ changes from negative to positive values. Hence, the particle changes direction at $t = 7$ and at $t \approx 8.5$.

(C) What is the acceleration at time, $t = 2$.

$$a(2) = v'(2) = \frac{v(3) - v(0)}{3 - 0}$$

$a(t) = v'(t)$ So,

$$a(2) = \frac{4}{3}$$

Be sure to remember how to do anything that we have done on any projects, puzzles, or quizzes for this chapter. [I want to save paper!]

Revised on 30 September 2009