More Chapters 6 and 7 Review

Solve the following separable differential equation with the initial condition $y(4)=0$

$$\frac{dy}{dx}=\frac{x}{-\sin y}$$

$$-\sin y \, dy = x \, dx$$

$$\int -\sin y \, dy = \int x \, dx$$

$$\cos y = \frac{x^2}{2} + C$$

$$\underline{\cos 0} = \frac{4^2}{2} + C \qquad C = -7$$

$$\cos y = \frac{x^2}{2} - 7$$

$$\cos^{-1}(\cos y) = \cos^{-1}\left(\frac{x^2}{2} - 7\right)$$

$$y = \cos^{-1}\left(\frac{x^2}{2} - 7\right)$$

Solve the following separable differential equation with the initial condition $y(1) = 2$

$$\frac{dy}{dx} = 4x^3 e^{-3y}$$

$$e^{3y}\,dy = 4x^3\,dx$$

$$\int e^{3y}\,dy = \int 4x^3\,dx$$

$$\frac{1}{3}e^{3y} = x^4 + C_1$$

$$e^{3y} = 3x^4 + C$$

$$e^6 = 3 + C$$

$$e^6 - 3 = C$$

$$e^{3y} = 3x^4 + e^6 - 3$$

$$\ln e^{3y} = \ln(3x^4 + e^6 - 3)$$

$$3y = \ln(3x^4 + e^6 - 3)$$

$$y = \frac{1}{3}\ln(3x^4 + e^6 - 3)$$

♪ $\frac{1}{e^{-3y}} = e^{3y}$

$$\boxed{\begin{array}{l} u = 3y \\ du = 3\,dy \\ \frac{1}{3}du = dy \\ \frac{1}{3}\int e^u\,du \end{array}}$$

♪ $3C_1 = C_2$

call it $= C$

Find the particular solution to the separable differential equation
$\frac{dy}{dx} = \frac{y-2}{2x}$ with the initial condition $f(e) = 3$

$$\frac{1}{y-2}\,dy = \frac{1}{2x}\,dx$$

$$\int \frac{1}{y-2}\,dy = \frac{1}{2}\int \frac{1}{x}\,dy$$

$$\ln|y-2| = \frac{1}{2}\ln|x| + C$$

$$\ln|3-2| = \frac{1}{2}\ln|e| + C$$

$$0 = \frac{1}{2} + C \qquad \text{Hence, } C = -\frac{1}{2}$$

$$\ln|y-2| = \frac{1}{2}\ln|x| - \frac{1}{2}$$

$$e^{\ln|y-2|} = e^{\frac{1}{2}\ln|x| - \frac{1}{2}}$$

$$|y-2| = e^{\frac{1}{2}\ln|x|} \cdot e^{-\frac{1}{2}}$$

$$y-2 = (\sqrt{x})\,e^{-\frac{1}{2}}$$

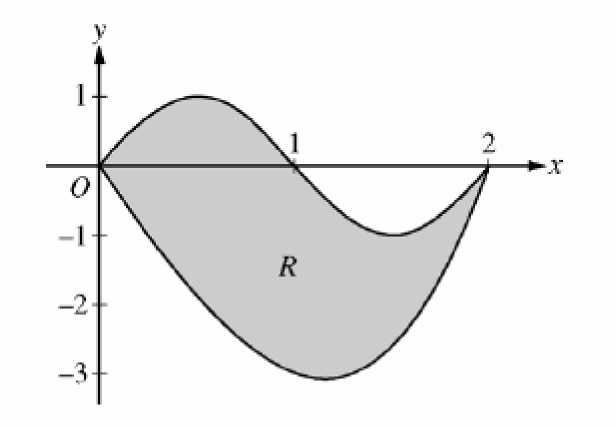
$$y = 2 + e^{-\frac{1}{2}}\sqrt{x}$$

CHECK

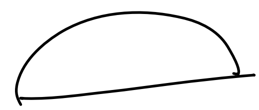
$$3 = 2 + e^{-\frac{1}{2}}\sqrt{e}$$

$$3 = 2 + e^{-\frac{1}{2}} \cdot e^{\frac{1}{2}} \checkmark$$

More fun with 2008AB1



$y = \sin(\pi x)$ and $y = x^3 - 4x$



(a) Let R be the base of a solid whose cross sections are semi-circles. For this solid, each cross section is perpendicular to the x-axis. Find the volume of this solid.

$A(x) = \frac{\pi}{2} r^2$

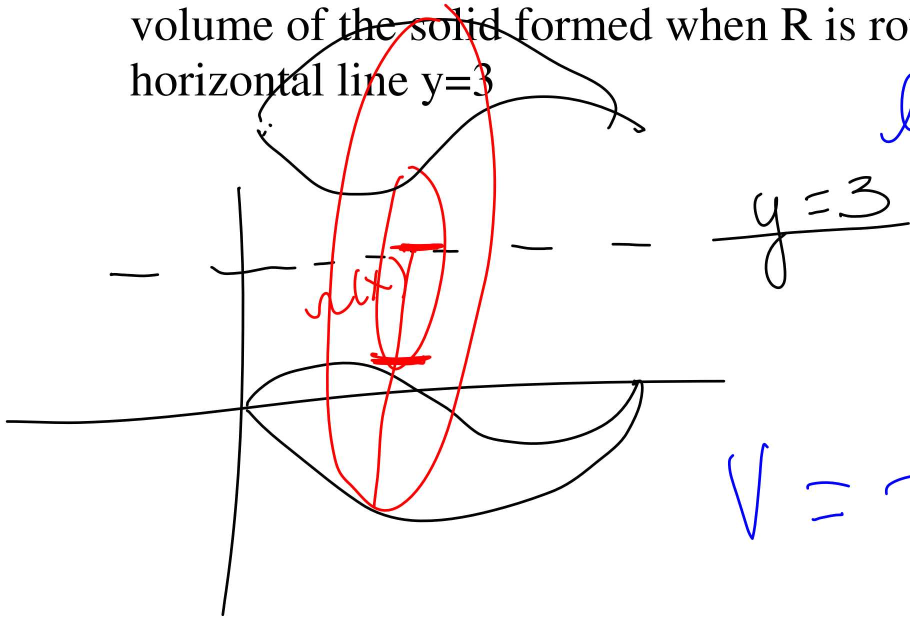
$r = \frac{1}{2}\left[\sin(\pi x) - (x^3-4x)\right]$

$V = \frac{\pi}{2}\int_0^2 \left[\frac{1}{2}\left(\sin(\pi x) - (x^3-4x)\right)\right]^2 dx$

$= \frac{\pi}{8}\int_0^2 \left((\sin \pi x) - (x^3-4x)\right]^2 dx$

$V \simeq 1.247\pi$ c.u.

(b) Set up, but do not integrate, an expression to find the volume of the solid formed when R is rotated about the horizontal line y=3



let $R(x) = 3 - (x^3 - 4x)$

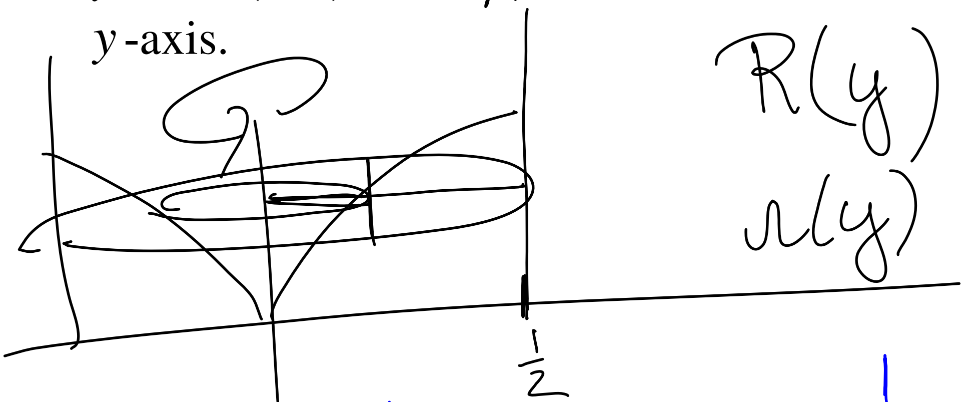
$r(x) = 3 - \sin(\pi x)$

$$V = \pi \int_0^2 \left[(R(x))^2 - (r(x))^2 \right] dx$$

(c) The vertical line x=k splits the region R into two parts of equal area. Write, but do not evaluate an integral expression to represent this. [Need to have "k"]

$$\int_0^k \left[\sin(\pi x) - (x^3 - 4x) \right] dx = 2$$

(d) Set up, but do not integrate, an expression to find the volume of the solid formed when the region bounded by $y = \sin(\pi x)$ on $[0, \frac{1}{2}]$ is rotated [revolved] about the y-axis.



$R(y) = \frac{1}{2}$

$r(y) = \frac{1}{\pi} \sin^{-1}(y)$

$y = \sin(\pi x)$

$\sin^{-1}(y) = \pi x$

$\frac{1}{\pi} \sin^{-1}(y) = x$

$$V = \pi \int_0^1 \left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{\pi} \sin^{-1}(y)\right)^2 \right] dy$$

2008 AB1B [calculator]

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$

(a) Find the area of R

TOP FUNCTION: $y = \sqrt{x}$ BOTTOM: $y = \frac{x}{3}$

LOWER LIMIT: $x = 0$ UPPER: $x = 9$

$$A = \int_0^9 \left[\sqrt{x} - \frac{x}{3}\right] dx = \boxed{4.5 = A}$$

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$

$$y = \frac{x}{3} + 1$$

$$3y + 1 = R(y)$$

$$y^2 + 1 = r(y)$$

$$V = \pi \int_0^3 \left[[R(y)]^2 - [r(y)]^2 \right] dy$$

$$V = 41.4\pi \text{ units}^3$$

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

$$A(y) = s^2$$

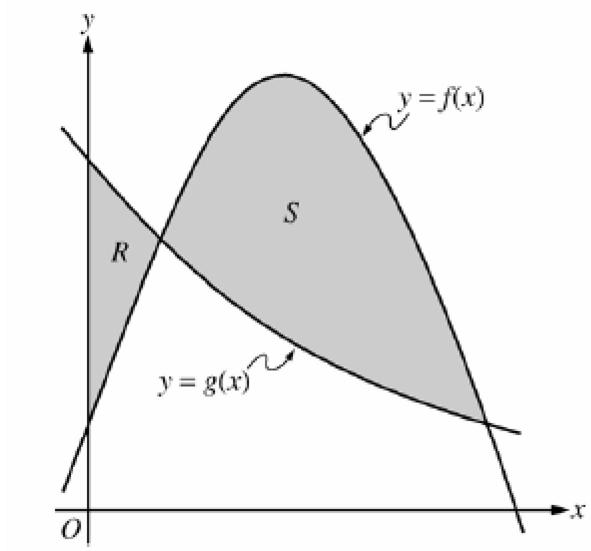
$$s = g(y) - f(y)$$

$$f(y) = y^2$$

$$g(y) = 3y$$

$$V = \int_0^3 [g(y) - f(y)]^2 dy \approx 8.100$$

2005 AB1 [calculator]



Let f and g be the functions given by $f(x)=\frac{1}{4}+\sin(\pi x)$ and $g(x)=4^{-x}$. Let R be the region in the first quadrant enclosed by the $y-$axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g as shown above.

(a) Find the area of R

$f(x) = g(x)$ at $.17821805$ let that equal A.

$$\text{Area of } R = \int_0^A (g(x) - f(x))\,dx \approx .065$$

(b) Find the area of S

f (x) top fx'n

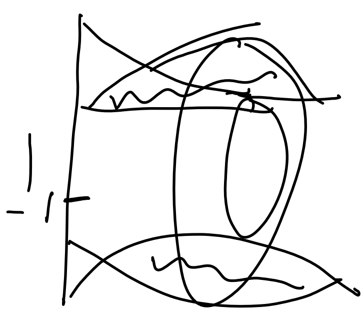
g (x) bottom

intersect $x=1$

$$A = \int_{A}^{1} \left[\left(\frac{1}{4} + \sin(\pi x) \right) - \left(4^{-x} \right) \right] dx$$

$$\boxed{= .410}$$

(c) Find the volume of the solid generated when s is revolved about the horizontal line $y = -1$



$R(x) = f(x) + 1$

$r(x) = g(x) + 1$

$$\text{Volume} = \pi \int_{A}^{1} \left[(R(x))^2 - (r(x))^2 \right] dx$$

$$\approx \boxed{1.452\pi \text{ un}^3}$$