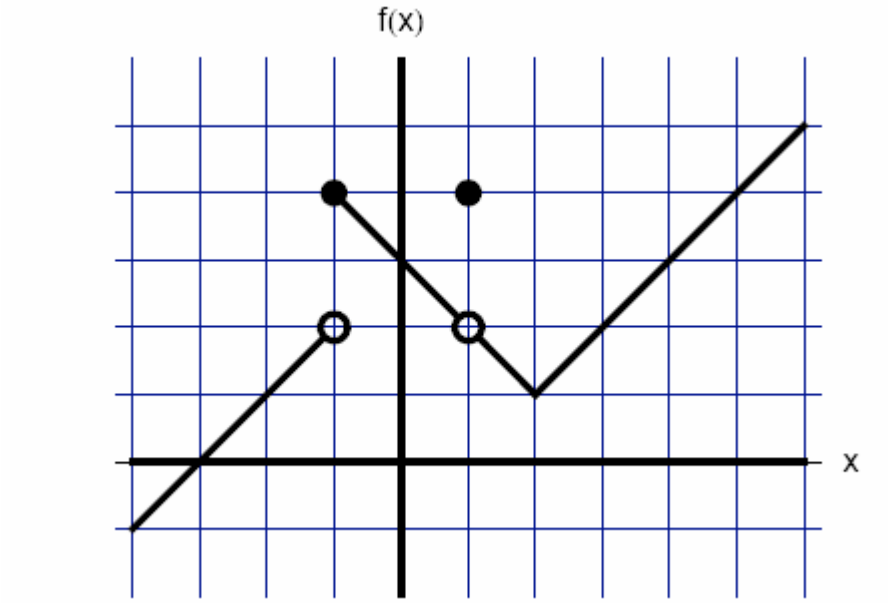
☺**My Chapter One AP Calculus AB Test** _____

[Limits and Continuity]

A graphing calculator is NOT permitted on this portion of the test

Directions: Solve each of the following problems using the available space for your scratch work. After examining the form of the choices, decide which is the best of the choices and fill in the corresponding bubble on your Scrantron form. No credit will be given for anything in your scratch work. Do not spend too much time on any one problem.



1. The graph of a function f whose domain is the closed interval $[-4, 6]$ is shown *above*. Which of the following statements about $f(x)$ is true?

(A) $\lim_{x \to -1} f(x) = 4$

(B) $\lim_{x \to -1} f(x) = 2$

(C) $f(x)$ is continuous at $x = -1$

(D) $f(x)$ is continuous at $x = 1$

(E) $\lim_{x \to 2} f(x) = f(2)$

2. Determine $\lim_{x \to 2} (x + 2)$

(A) 2
(B) 0
(C) 4
(D) 7
(E) Does not exist

3. Determine $\lim_{x \to \frac{\pi}{4}} \sin x$

(A) 0
(B) 1
(C) $\frac{\sqrt{2}}{2}$
(D) $\sqrt{2}$
(E) $\frac{\sqrt{3}}{2}$

4. Let $f(x) = \begin{cases} x^2 + 4, & x \neq 1 \\ 1, & x = 1 \end{cases}$ and determine $\lim_{x \to 1} f(x)$

(A) 5
(B) 1
(C) 4
(D) 16
(E) Does not exist

5. Determine $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ where $a \neq 0$

(A) $2a^2$
(B) $-2a^2$
(C) $\frac{-1}{2a^2}$
(D) $\frac{1}{2a^2}$
(E) Does not exist

6. Determine $\lim_{x \to 4} \dfrac{x^2 + 2x - 24}{x - 4}$

(A) 10
(B) 6
(C) -6
(D) 4
(E) Does not exist

7. If f is a continuous function defined by $f(x) = \begin{cases} x^2 + bx, & x \le 1 \\ 5\sin\left(\dfrac{\pi}{2}x\right), & x > 1 \end{cases}$ then $b =$

(A) -4
(B) 4
(C) 5
(D) -5
(E) 10

8. If $\lim_{x \to 2} f(x) = 3$ and $\lim_{x \to 2} g(x) = 5$ then $\lim_{x \to 2} [f(x) + g(x)] =$

(A) 4
(B) 0
(C) -2
(D) 8
(E) $\dfrac{3}{5}$

9. Determine $\lim_{x \to 5} \dfrac{x^3 - 15x^2 + 75x - 125}{x - 5}$

(A) Does not exist
(B) 0
(C) 1
(D) 25
(E) 125

10. Determine $\lim_{x \to 1} \frac{\ln x + 3x}{x}$

(A) 0
(B) 1
(C) e
(D) 2
(E) 3

11. Determine $\lim_{x \to e} \ln x^2$

(A) 0
(B) 1
(C) e
(D) 2
(E) 3

12. Determine $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ when $f(x) = 3x^2$

(A) Does not exist
(B) 0
(C) x
(D) 6x
(E) 9x

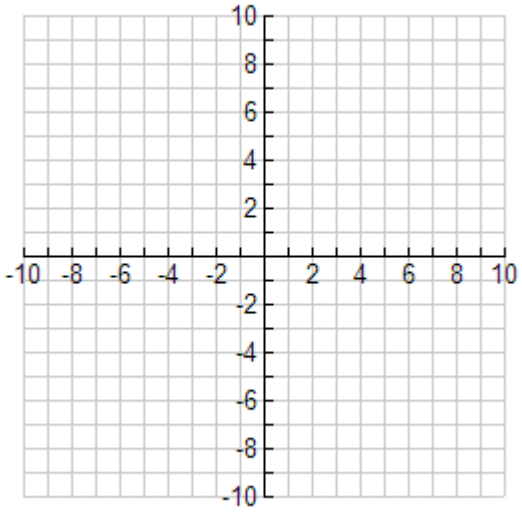
FREE RESPONSE
Show all work in the space provided. All steps must be shown. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as the accuracy of your final answers. Correct answers without supporting work ["bald" answers] will NOT receive credit. Justifications require mathematical [non-calculator] reasons. Your work must be expressed in standard mathematical notation. Unless otherwise specified, answers [numeric or algebraic] need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

FR1. On the axes provided below, sketch a graph of a function that has all of the following attributes listed below:

I. $\lim_{x \to 3} f(x) = 4$

II. $f(3) = -2$

III. $\lim_{x \to -3} f(x) = \infty$



FR2. $f(x)$ and $g(x)$ are continuous functions for all $x \in$ Reals. The table below has values for the functions for selected values of x. The function $h(x) = g(f(x)) + 2$.

x	$f(x)$	$g(x)$
1	3	4
3	9	-10
5	7	5
7	11	25

Explain why there must be a value c for $1 < c < 5$ such that $h(c) = 0$.

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A GRAPHING CALCULATOR MAY BE USED ON THIS PART OF THE TEST

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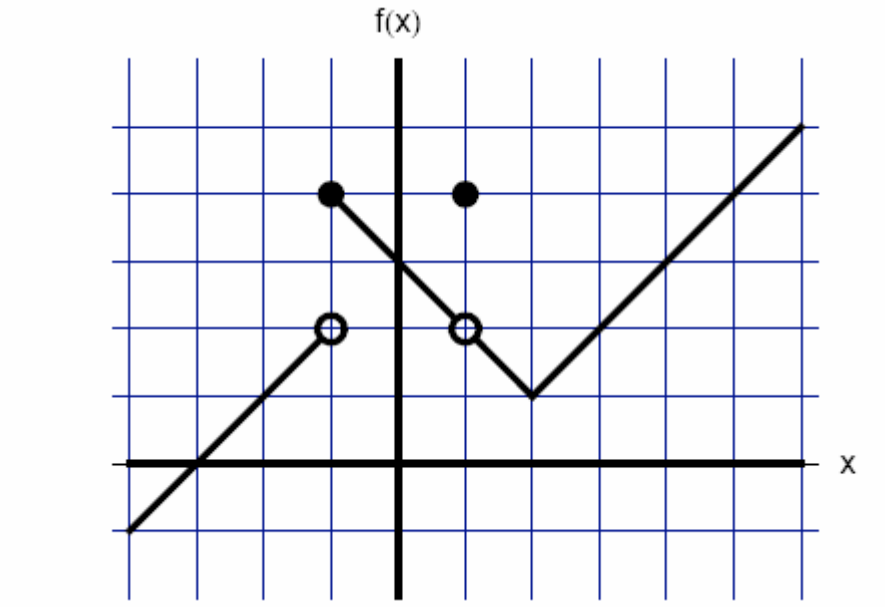

13. The graph of a function f is shown above. Which of the following statements about f is false?

(A) $\lim_{x \to -1^+} f(x) = 4$

(B) $\lim_{x \to -1^-} f(x) = 2$

(C) $\lim_{x \to 1} f(x) = 4$

(D) $\lim_{x \to 1} f(x) = 2$

(E) f is continuous at $x = 2$

14. Determine $\lim_{x \to 3} \dfrac{|x-2|}{x-2}$

(A) 1 (B) 0 (C) -1 (D) 3 (E) Does not exist

15. Determine $\lim_{x \to 2} \dfrac{|x-2|}{x-2}$

(A) 1 (B) 0 (C) -1 (D) 3 (E) Does not exist

16. Consider the function $f(x) = \begin{cases} \dfrac{1-\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$.

In order for $f(x)$ to be continuous at $x = 0$, then the value of k must be

(A) 1
(B) 0
(C) π
(D) -1
(E) Can not be made continuous at $x = 0$

17. A function $g(x)$ has a vertical asymptote at $x = 3$. Which of the following statement(s) must be true about $g(x)$?

I. $g(x)$ is continuous at $x = 3$
II. $\lim_{x \to 3} g(x) = \infty$
III. $\lim_{x \to -3} g(x) = \infty$

(A) I, II, and III
(B) I only
(C) II only
(D) I and II
(E) II and III

18. Determine $\lim_{x \to 1} \dfrac{\cos(x-1)}{x-1}$

(A) Does not exist
(B) 0
(C) 1
(D) π
(E) $\dfrac{1}{\sqrt{2}}$

19. Determine $\lim_{x \to 0} \sin\left(\dfrac{1}{x}\right)$

(A) 0
(B) 1
(C) Does not exist
(D) π
(E) -1

20. If f is continuous on $[-2,4]$ and $f(-2) = 5$, $f(0) = -3$, *and* $f(4) = 711$, then according to the Intermediate Value Theorem, how many zeroes are guaranteed on the closed interval $[-2, 4]$

(A) none
(B) one
(C) two
(D) three
(E) four

FREE RESPONSE
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FR3. Find a such that the function $f(x)=\begin{cases}\dfrac{4\sin x}{x}, & x<0\\ a+15x, & x\geq 0\end{cases}$ is continuous for all real numbers.

BONUS

Let f be the function defined by

$$f(x)=\begin{cases}\sqrt{x+1} & \text{for } 0\leq x\leq 3\\ 5-x & \text{for } 3<x\leq 5.\end{cases}$$

Is f continuous at $x=3$? Explain why or why not.

Calculus Word Search [Just for fun!]
From: http://www.free-online-word-search-puzzles.com/calculus.htm

Find the words that have been hidden in the grid, then read a statement by a famous philosopher and mathematician.

M C U R V E A E X P O N E N T I A L A
T L O G A R I T H M H E M A T I C R C
I N E N I S O C T N A C E S S O B R I
N E V G N L A R G E T N I R O E H T M
T G I L D L Y M U S V I O R G E M A U
E A T N E I T O U Q W O D L V U L L M
R T A E P N Y D P N T I A A M A P U I
C I V O E E S R O S N E R I I O C M N
E V I S N A N I T A S I X T W A E R I
P E R S D R T A T E A A N E L A N O M
T E E O E C T E T B M E R C H N O F N
L P D Y N T Q C L U R O U R P G U T P
T O H U T U O E B E R L N U A L N R S
T L F S A N S U F P U A R O R E O E U
M S E T S Q B F E S A U L A G D S T R
N O I T U T I T S B U S T N U I Y B F
E O A A R D T R A N D E A C G R R U A
N N R D E F I N I T E T T N S S E T C
T E L L A E R A E C N E R E F F I D E

Algebra	Formula	Quotient
Angle	Function	Rate
Area	Graph	Root
Calculus	Independent	Secant
Constant	Integral	Sign
Coordinate	Intercept	Slope
Cosine	Linear	Square
Curve	Logarithm	Substitution
Definite	Maximum	Sum
Derivative	Minimum	Surface
Difference	Natural	Tangent
Differential	Negative	Trigonometry
Equation	Power	Variable
Exponential	Product	