My Chapter Four Riemann Handout

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

   (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

   (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

   (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

   (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

   (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

   (a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

   (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

The rate at which water is being pumped into a tank is given by the continuous, increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 \le t \le 20$ minutes, is shown below.

t (min)	0	4	9	17	20
$R(t)$ (gal/min)	25	28	33	42	46

a. Use a right Riemann sum with four subintervals to approximate the value of:

$$\int_0^{20} R(t)\,dt.$$

the APPROX # of gallons PUMPED IN during $0 \le t \le 20$ MINUTES

Is your approximation greater or less than the true value? Give a reason for your answer.

$RRAM = 4R(4) + 5R(9) + 8R(17) + 3R(20)$
$= 4(28) + 5(33) + 8(42) + 3(46)$
$= 751$ gallons

SINCE $R(t)$ IS INCREASING ON $0 \le t \le 20$ then we HAVE AN OVER-APPROXIMATION [RRAM > ACTUAL]

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Car A has positive velocity $v_A(t)$ as it travels on a straight road, where v_A is a differentiable function of t. The velocity is recorded for selected values over the time interval $0 \le t \le 10$ seconds, as shown in the table below.

t (sec)	0	2	5	7	10
$v_A(t)$ (ft/sec)	0	9	36	61	115

b. Use data from the table to approximate the distance traveled by Car A over the interval $0 \le t \le 10$ seconds by using a ~~trapezoidal~~ sum with four subintervals. Show the computations that lead to your answer, and indicate units of measure.

Let's do LRAM and RRAM instead!

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

A particle moves along a horizontal line with a positive velocity $v(t)$, where v is a differentiable function of t. The time t is measured in seconds, and the velocity is measured in cm/sec. The velocity of the particle at selected times is given in the table below.

t (sec)	0	2	4	6	8	10	12
$v(t)$(cm/sec)	37	17	5	1	6	17	38

Use a midpoint Riemann sum with three subintervals of equal length and values from the table to approximate:

$$\int_0^{12} v(t)\,dt.$$

Show the computations that lead to your answer. Using correct units, explain the meaning of this definite integral in terms of the particle's motion.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

   (a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

AVG ACC on $[0,80]$ $= \dfrac{v(80)-v(0)}{80-0}$

$= \dfrac{49-5}{80}$

$= .55 \dfrac{ft}{sec^2}$

$\int_{10}^{70} v(t)\,dt \approx$ MRAM

MRAM $= 20[v(20)+v(40)+v(60)]$

$= 20[22+35+44]$

$= 2020$ ft

Since $v(t) > 0$ on $10 \le t \le 70$ sec

Then $\int_{10}^{70} v(t)\,dt$ gives us an approx. of total distance traveled in feet of the rocket during $10 \le t \le 70$ sec

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

$\int_0^{24} R(t)\,dt \approx MRAM$

$MRAM = 6[R(3) + R(9) + R(15) + R(21)]$

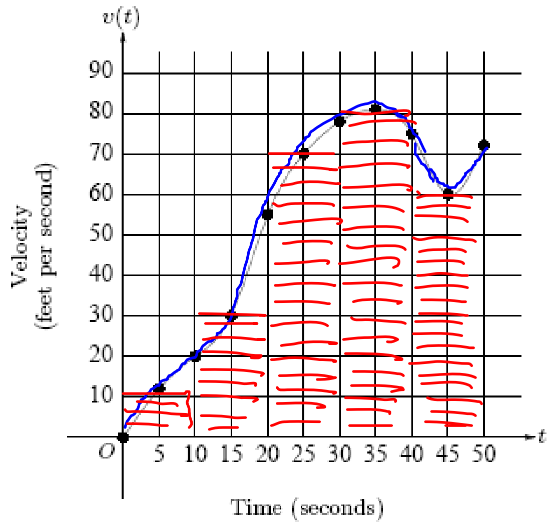
$= 6[10.4 + 11.2 + 11.3 + 10.2]$

$= 258.6$ gallons

$\int_0^{24} R(t)\,dt$ gives us an approximation of the # of gallons pumped out during $0 \le t \le 24$ hours

By MVT there is a t, $0 < t < 24$, such that $R'(t) = \frac{R(24) - R(0)}{24 - 0}$

Since $\frac{R(24) - R(0)}{24} = 0$ then there is a t, $0 < t < 24$ such that $R'(t) = 0$



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

Approximate $\int_0^{50} v(t)\,dt$ with a Riemann Sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

$\int_0^{50} v(t)\,dt \approx \text{MRAM}$

$\text{MRAM} = 10\,[v(5) + v(15) + v(25) + v(35) + v(45)]$

$= 10\,[12 + 30 + 70 + 81 + 60]$

$= 2530 \text{ ft}$

Since $v(t) > 0$ on $0 \le t \le 50$, then $\int_0^{50} v(t)\,dt$ gives us the TOTAL DISTANCE TRAVELED IN FEET BY THE CAR ON $0 \le t \le 50$ sec

IF YOU WERE NOT IN CLASS, THEN DO THE STARRED PROBLEMS FOR HOMEWORK