My Free Response Part of AP Calculus AB First Semester Exam [December 2008]

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FR1

If $f(x)$ and $g(x)$ are twice differentiable and if $h(x) = f(g(x))$, then find $h'(x)$.

$$h(x) = f(g(x))$$

Since $h(x)$ is a composite function, then we will need to use the Chain Rule. We can let $u = g(x)$ and $\frac{du}{dx} = g'(x)$

So, $h'(x) = f'(u)\frac{du}{dx}$

Hence, $h'(x) = f'(g(x))\,g'(x)$

FR2

Ms. McCleary's rocket ship has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 70$ seconds as shown in the table below.

t Seconds	0	10	20	30	40	50	60	70
$v(t)$ Meters per second	5	14	22	29	35	40	35	50

(a) Show that there is at least one time during the recorded flight time that the acceleration of the rocket is equal to zero.

By the Mean Value Theorem there is a t, $40 < t < 60$, which is contained in $0 < t < 60$, such that $v'(t) = a(t) = \dfrac{v(60) - v(40)}{60 - 40}$ which equals 0. Hence, there is at least one time during the recorded flight that the acceleration of the rocket is equal to zero.

(b) Using correct units, explain the meaning of $\int_0^{70} v(t)\,dt$ in terms of the rocket's flight. Then use a right Riemann sum to approximate the definite integral.

Since $v(t) > 0$ on $0 \le t \le 70$, then $\int_0^{70} v(t)\,dt$ will give us the total distance traveled in meters by the rocket during the time interval of $0 \le t \le 70$ seconds.

$$\int_0^{70} v(t)\,dt \approx RRAM$$

$$RRAM = 10\,[v(10) + v(20) + v(30) + v(40) + v(50) + v(60) + v(70)]$$

$$= 10\,[14+22+29+35+40+35+50]$$

$$= 2250 \text{ meters}$$