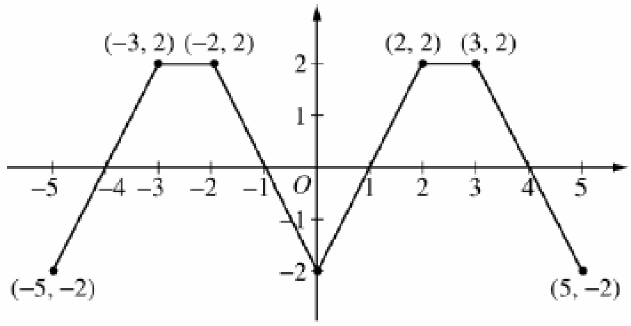
Chapter Four Handout

The following questions are the property of ETS College Board
2006 AB 3 [calculator]



Graph of f

3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

2007 AB 5 [non-calculator]

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

2006 AB 4 [non-calculator]

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

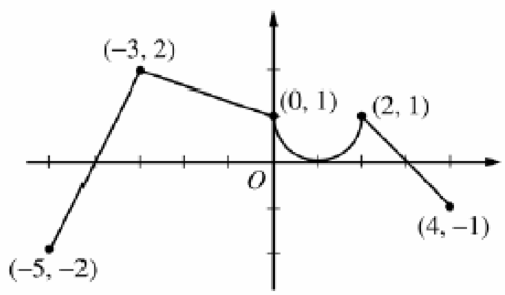
4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

   (a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

   (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

2004 AB5 [non-calculator]



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

   (a) Find $g(0)$ and $g'(0)$.

   (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

   (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

   (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

2005 AB3 [calculator]

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

   (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

   (c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

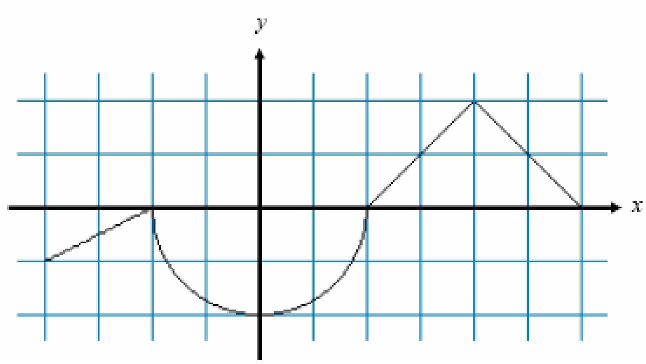
   (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

The following example is from: http://chaoticgolf.com

Example: The graph of f shown below consists of line segments and a semicircle. Evaluate each definite integral.

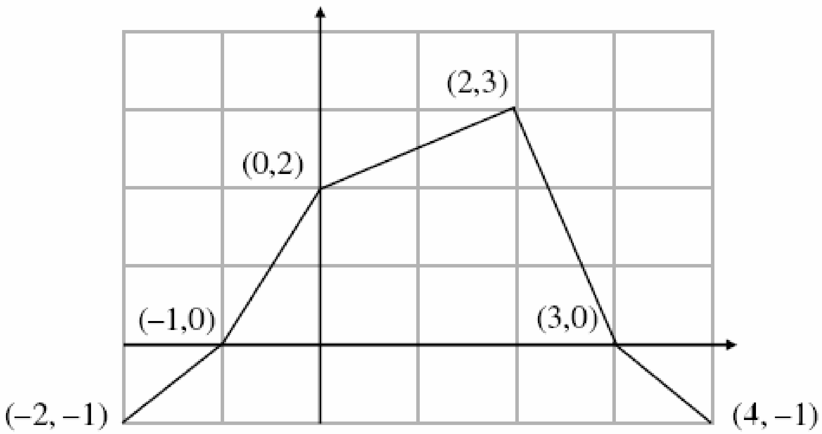
a) $\int_0^2 f(x)\,dx$ b) $\int_2^6 f(x)\,dx$ c) $\int_{-4}^2 f(x)\,dx$

d) $\int_{-4}^6 f(x)\,dx$ e) $\int_{-4}^6 |f(x)|\,dx$ f) $\int_{-4}^6 [f(x)+2]\,dx$



The following question is from: http://www.houstonact.org

The Detective's Hat Function



The graph of the function f shown above is a piecewise continuous function defined on $[-2, 4]$. The graph of f consists of five line segments.

Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

1. Find each of the following.

   (a) $g(-2)$ (b) $g(-1)$ (c) $g(0)$ (d) $g(2)$ (e) $g(3)$ (f) $g(4)$

2. Explain the procedure you followed to answer question 1.

3. Find each of the following.

   (a) $g'(-1)$ (b) $g'(0)$ (c) $g'(2)$ (d) $g'(4)$

4. Explain the procedure you followed to answer question 3.

5. Explain why g must be a continuous function on $[-2, 4]$.

6. Write the equation for $g'(x)$ on the interval $[0, 2]$.

7. Write the equation for the line tangent to g at $x = 1$. Justify your answer.

8. Does $g''(0)$ exist? Explain your reasoning.

9. Will a point of inflection for g exist when $x = 0$? Explain your reasoning.

10. For what values of x in the open interval $(-2, 4)$ is g increasing? Explain your reasoning.

11. For what values of x in the open interval $(-2, 4)$ is g decreasing?

12. For what values of x in the open interval $(-2, 4)$ is g concave up? Explain your reasoning.

13. For what values of x in the open interval $(-2, 4)$ is g concave down?

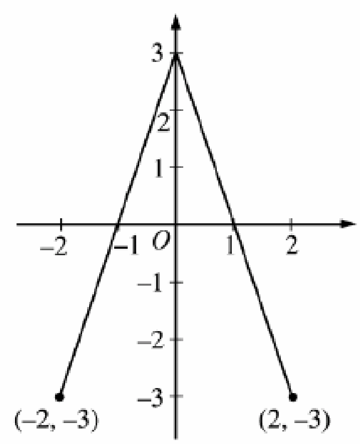
14. Find the maximum and the minimum values of g on the closed interval $[-2, 4]$. Justify your answers.

Another AP Question from ETS College Board
2002 AB4

AP Calculus AB-4 / BC-4 Final Draft for Scoring 2002

The graph of the function f shown above consists of two line segments. Let g be the function given by

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.
(b) For what values of x in the open interval (-2, 2) is g increasing? Explain your reasoning.
(c) For what values of x in the open interval (-2,2) is the graph of g concave down? Explain your reasoning.
(d) Sketch the graph of g on the closed interval [-2, 2].



Graph of f

Some AP or AP-type Multiple-Choice questions which I found at:
http://www.houstonact.org/documents/WoodFunctions.pdf [any relation to Erica or Jacquie?]

24. Let $f(x) = \int_0^x \frac{t^2 - 4}{1 + \cos^2 t} dt$. At what value of x does the local maximum of $f(x)$ occur?

A) –4 B) –3 C) –2 D) –1

E) 0 F) 1 G) 2 H) 3

25. Let $y = \int_1^{3x} \frac{dt}{t^2 + t + 1}$. Find $\frac{d^2y}{dx^2}$.

A) $\frac{(18x+3)}{(9x^2+3x+1)^2}$

B) $\frac{3(18x+3)}{(9x^2+3x+1)^2}$

C) $\frac{-3(18x+3)}{(9x^2+3x+1)^2}$

D) $\frac{3(18x+3)}{(9x^2+3x+1)^3}$

E) $\frac{(18x+3)}{(9x^2+3x+1)^2}$

F) $\frac{(18x+3)}{(9x^2+3x+1)^3}$

G) $\frac{(18x+3)^2}{(9x^2+3x+1)^3}$

H) $\frac{-3(18x+3)^2}{(9x^2+3x+1)^2}$

Example 30: (1998 AB multiple choice)

What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

(a) –3 (b) 0 (c) 3 (d) –3 and 3 (e) –3,0, and 3

19. 1997 BC Multiple Choice:

If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1) = 0$, the $f(4) =$

(A) –0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629