

My Chapter Three Test \_\_\_\_\_

**Directions:**

Solve each of the following problems. After examining the form of the choices, decide which is the best solution and fill in the corresponding "bubble" on your Scantron form. Be sure that your name is on both this page and on your Scantron form. Do not spend too much time on any one problem.

**NO CALCULATOR MAY BE USED ON THIS PART OF THE TEST**

1.

The function  $f$  given by  $f(x) = 2x^3 - 3x^2 - 12x$  has a relative minimum at  $x =$

- (A) -1      (B) 0      (C) 2      (D)  $\frac{3 - \sqrt{105}}{4}$       (E)  $\frac{3 + \sqrt{105}}{4}$

$$f'(x) = 6x^2 - 6x - 12$$
$$0 = 6(x-2)(x+1) \quad \text{c.v. at } x = -1, 2$$

$$f''(x) = 12x - 6$$

Second  
Der  
Test

$$f''(-1) < 0 \quad \text{rel max at } x = -1$$

$$f''(2) > 0 \quad \text{rel min at } x = 2$$

2.

For the function  $f$ ,  $f'(x) = 2x + 1$  and  $f(1) = 4$ . What is the approximation for  $f(1.2)$  found by using the line tangent to the graph of  $f$  at  $x = 1$ ?

- (A) 0.6      (B) 3.4      (C) 4.2      (D) 4.6      (E) 4.64

eq of tan line at  $x = 1$

$$f'(1) = 3 \quad f(1) = 4$$

$$y - 4 = 3(x - 1)$$

let  $x = 1.2$

$$y - 4 = 3(1.2 - 1)$$
$$y = 4 + 3(0.2)$$

$$y = 4.6$$

3.

Let  $f$  be the function given by  $f(x) = x^3 - 6x^2$ . The graph of  $f$  is concave up when

- (A)  $x > 2$
- (B)  $x < 2$
- (C)  $0 < x < 4$
- (D)  $x < 0$  or  $x > 4$  only
- (E)  $x > 6$  only

$$\begin{aligned} f'(x) &= 3x^2 - 12x \\ f''(x) &= 6x - 12 \\ \text{let } 0 &= 6x - 12 \quad x = 2 \\ &(-\infty, 2) \quad (2, \infty) \\ &f''(x) < 0 \quad f''(x) > 0 \end{aligned}$$

4. Find:  $\lim_{x \rightarrow \infty} \frac{5x^3 - 3x + 7}{7 + 9x^2 - 10x^3}$

- (A) 7
- (B) 5
- (C)  $\frac{1}{2}$
- (D)  $-\frac{1}{2}$
- (E) Does not exist

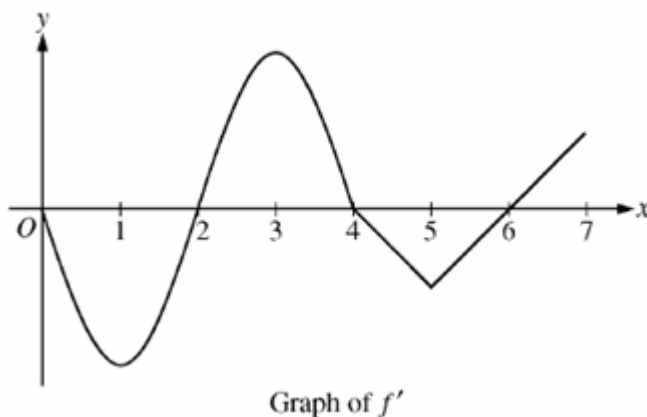
$$\lim_{x \rightarrow \infty} \frac{5x^3}{-10x^3} = -\frac{1}{2}$$

5. Find:  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 25}}{3x + 7}$

- (A)  $\frac{5}{7}$
- (B)  $-\frac{1}{3}$
- (C)  $\frac{1}{3}$
- (D)  $\frac{25}{7}$
- (E) Does not exist

$$\lim_{x \rightarrow -\infty} \frac{-x}{3x} = -\frac{1}{3}$$

6. Let  $f$  be the function whose derivative is given in the graph below. Which of the following statements about  $f$  are FALSE?



GRAPH of  $f'$

- (A) The graph of  $f$  has critical values at  $x=0$ ,  $x=2$ ,  $x=4$ ,  $x=6$   
 (B) The graph of  $f$  is increasing on  $(2, 4)$  and  $(6, 7)$   
 (C) The graph of  $f$  has points of inflection at  $x=1$ ,  $x=3$ ,  $x=5$   
 (D) The graph of  $f$  has a relative minima at  $x=1$  and  $x=5$   
 (E) The graph of  $f$  has a relative maximum at  $x=4$

7. Which of the following statements is true for  $f(x) = 1 + \sqrt[3]{x}$ ?

- I  $f(x)$  is always increasing for all  $x \neq 0$  *yes*  
 II The tangent to the curve at  $x=0$  is horizontal *NO*  
 III The Mean Value Theorem can be applied to  $f(x)$  on  $[-1, 1]$  *NO [NOT DIFF at  $x=0$ ]*

- (A) I only  
 (B) II only  
 (C) III only  
 (D) II and III only  
 (E) I, II, and III

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \quad \text{C.V. at } x=0$$

$$f(1) = 2$$

$$f(-1) = 0$$

**Free Response Directions**

Write clearly and legibly using standard mathematical notation. Show all work/steps. Correct answers without supporting work will receive NO credit. Unless otherwise specified, answers [numeric or algebraic] need not be simplified. If your answer is given as a decimal approximation, it should be correct to THREE decimal places. You are permitted [on Part II] to use your calculator to solve an equation, find a numerical derivative or calculate a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. Justifications required mathematical [non-calculator] reasons.

**FR 1**

$t$ [in days]	$W(t)$ [in $^{\circ}F$ ]
0	50
3	61
6	58
9	54
12	50
15	55

The temperature in degrees Fahrenheit ( $^{\circ}F$ ), of the water in Ms. McCleary's pond is a differentiable function  $W$  of time  $t$ . The table above shows the water temperature as recorded every 3 days over a 15-day period.

- (a) Use data from the table to find an approximation for  $W'(6)$ . Show the computations that lead to your answer. Indicate units of measure.
- (b) For  $0 < t < 15$ , must there be a time when  $W'(t) = 0$ ? Justify your answer [with Calculus].

***End of Part I. Turn in this portion of the test and pick up Part II. Hold onto your Scantron. You may not return to this portion of the test once you have turned it in.***

**My Chapter Three Test** \_\_\_\_\_

Solve each of the following problems. After examining the form of the choices, decide which is the best solution and fill in the corresponding “bubble” on your Scantron form. Be sure that your name is on both this page and on your Scantron form. Do not spend too much time on any one problem.

**A GRAPHING CALCULATOR MAY BE NECESSARY FOR SOME PROBLEMS ON THIS PART OF THE TEST**

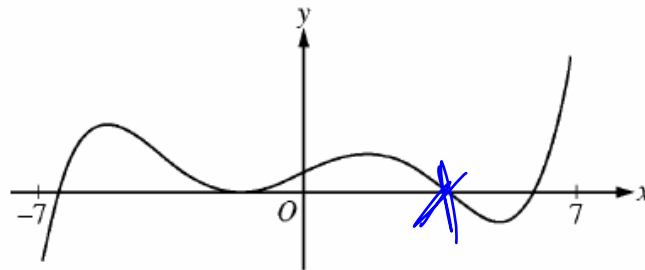
8.

Let  $f$  be the function with first derivative defined by  $f'(x) = \sin(x^3)$  for  $0 \leq x \leq 2$ . At what value of  $x$  does  $f$  attain its maximum value on the closed interval  $0 \leq x \leq 2$ ?

- (A) 0      (B) 1.162      (C) 1.465      (D) 1.845      (E) 2

*use TI to graph  $f'(x)$   
 $f'(x)$  changes from  
positive to negative  
value at  $x \approx 1.465$*

9.



Graph of  $f'$

The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , on the open interval  $-7 < x < 7$ . If  $f'$  has four zeros on  $-7 < x < 7$ , how many relative maxima does  $f$  have on  $-7 < x < 7$ ?

- (A) One      (B) Two      (C) Three      (D) Four      (E) Five

*$f'(x)$  changes from  
positive to negative  
only once*

10.

$x$	3	4	5	6	7
$f(x)$	20	17	12	16	20

The function  $f$  is continuous and differentiable on the closed interval  $[3, 7]$ . The table above gives selected values of  $f$  on this interval. Which of the following statements must be true?

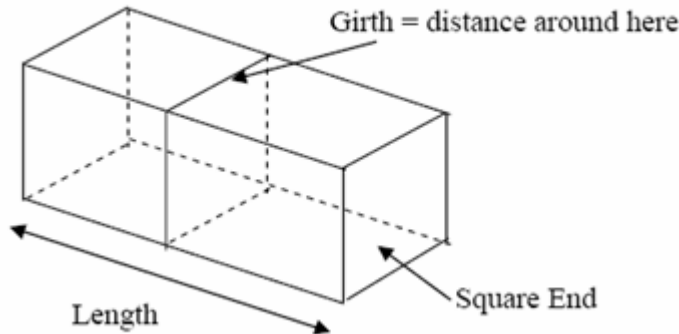
- I. The minimum value of  $f$  on  $[3, 7]$  is 12. *UNKNOWN*
  - II. There exists  $c$ , for  $3 < c < 7$ , such that  $f'(c) = 0$ . *yes MVT OR ROLLÉ'S*
  - III.  $f'(x) > 0$  for  $5 < x < 7$ . *UNKNOWN*
- (A) I only  
(B) II only  
(C) III only  
(D) I and III only  
(E) I, II, and III

11.

If  $f$  is a continuous function on the closed interval  $[a, b]$ , which of the following must be true?

- DOES NOT SAY IF  $f$  IS DIFFERENTIABLE*
- (A) There is a number  $c$  in the open interval  $(a, b)$  such that  $f(c) = 0$ .
- (B) There is a number  $c$  in the open interval  $(a, b)$  such that  $f(a) < f(c) < f(b)$ .
- (C) There is a number  $c$  in the closed interval  $[a, b]$  such that  $f(c) \geq f(x)$  for all  $x$  in  $[a, b]$ . *S.U.T.*
- (D) There is a number  $c$  in the open interval  $(a, b)$  such that  $f'(c) = 0$ .
- (E) There is a number  $c$  in the open interval  $(a, b)$  such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

12. The Post Office of the country of Freedonia will accept a box for shipment only if the sum of its length and girth does not exceed 300 cm. Find the maximum volume possible.



- (A) 250000 cubic cm  
 (B) 100000 cubic cm  
 (C) 200000 cubic cm  
 (D) 300000 cubic cm  
 (E) 350000 cubic cm

$$V = lwh$$

$$V = x^2 l$$

$$300 = 4x + l$$

$$300 - 4x = l$$

$$V(x) = x^2(300 - 4x)$$

$$V(x) = 300x^2 - 4x^3$$

$$V'(x) = 600x - 12x^2$$

C.V. at  $x = 50$

$V'(x)$  changes from positive to negative at  $x = 50$

Bonus Multiple Choice [mark as #13 on your Scantron]

$x$	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

dim. 50 by 50 by 100  
 $V = 250000$

The function  $f$  is differentiable and has values as shown in the table above. Both  $f$  and  $f'$  are strictly increasing on the interval  $0 \leq x \leq 5$ . Which of the following could be the value of  $f'(3)$ ?

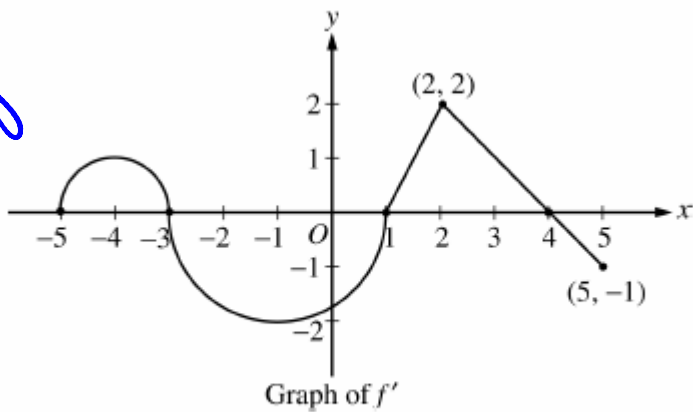
- (A) 20      (B) 27.5      (C) 29      (D) 30      (E) 30.5

### Free Response Directions

Write clearly and legibly using standard mathematical notation. Show all work/steps. Correct answers without supporting work will receive NO credit. Unless otherwise specified, answers [numeric or algebraic] need not be simplified. If your answer is given as a decimal approximation, it should be correct to THREE decimal places. You are permitted [on Part II] to use your calculator to solve an equation, find a numerical derivative or calculate a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. Justifications required mathematical [non-calculator] reasons.

### FR2

GRAPH of  $f'$



Let  $f$  be a function defined on the closed interval  $[-5, 5]$ . The graph of  $f'$ , the derivative of  $f$  is shown above.

(a) For  $-5 < x < 5$  find all  $x$  values at which  $f$  has a relative minimum. [Justify]

(b) For  $-5 < x < 5$  find all  $x$  values at which  $f$  has a point of inflection. [Justify]

(c) Find all intervals on which the graph of  $f$  is concave down. [Justify]